Analysis Seminar

Well-posedness and asymptotic behavior of one-dimensional wave equations with a set-valued boundary condition

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Date

Time: 10:00 am

On-line at Zoom

Abstract.

Wave equations with nonlinear boundary conditions have been the subject of several works in the past decades due to their importance from both theoretical and applied points of view (see, e.g., [1, 3, 4, 5]). From the perspective of control theory, nonlinear boundary conditions often arise from nonlinear phenomena in the practical implementation of boundary control laws for linear wave equations, such as nonlinearities in the components used for the implementation or saturation phenomena, and they may have an important impact in the stability properties and the asymptotic behavior of the system.

In this talk, after providing a brief summary of some important previous works on wave equations with nonlinear boundary damping, we will present a new framework for addressing this problem. We shall consider wave equations in L^p functional spaces and with set-valued boundary dampings, which are a natural generalization of nonlinear dampings allowing to fully exploit some symmetry properties previously observed and for which we can provide some very general well-posedness results.

We will show how our techniques allow us to retrieve some known results on the asymptotic behavior of wave equations with nonlinear boundary damping and provide answers to previously open questions. In particular, we provide a description of the decay rate of solutions for several nonlinear boundary conditions and we completely characterize the asymptotic behavior in the case of a boundary condition described by the sign function.

This talk is based on [2] and on ongoing joint works with Yacine Chitour and Swann Marx.

References

[1] F. Alabau-Boussouira. On some recent advances on stabilization for hyperbolic equations. In *Control of partial differential equations*, volume 2048 of *Lecture Notes in Math.*, pages 1–100. Springer, Heidelberg, 2012.

[2] Y. Chitour, S. Marx, and G. Mazanti. One-dimensional wave equation with set-valued boundary damping: well-posedness, asymptotic stability, and decay rates. *ESAIM Control Optim. Calc. Var*, in press.

[3] A. Haraux. L^p estimates of solutions to some non-linear wave equations in one space dimension. *International Journal of Mathematical Modelling and Numerical Optimisation*, 1(1-2):146–152, 2009.

[4] M. Pierre and J. Vancostenoble. Strong decay for one-dimensional wave equations with nonmonotone boundary damping. *Control and Cybernetics*, 29(2):473–484, 2000.

[5] J. Vancostenoble and P. Martinez. Optimality of energy estimates for the wave equation with nonlinear boundary velocity feedbacks. *SIAM J. Control Optim.*, 39(3):776–797, 2000.