**Problem set 1**. *Representação de grupos 1, 2020-2.* This counts as 15 percent of the grade. The second problem set will count as 15 percent of the grade. The written exam (online) will count as 70 percent of the grade.

## 1. Problem 1 (7 points)

Consider the following group: $G = S_3 \wr S_2$. This is the wreath product of $S_3$ and $S_2$, that is the semidirect product $(S_3 \times S_3) \rtimes S_2$ where, writing $S_2 = \langle \varepsilon \rangle$, where $\varepsilon = (12)$, the action is given by

$$(x, y)^{\varepsilon} := (y, x)$$

for every $x, y \in S_3$. This group has order $|G| = 2 \cdot 6^2 = 72 = 2^3 \cdot 3^2$ and it has 9 conjugacy classes. Find representatives for them and compute the character table of $G$.

Hint: $G$ acts naturally on $\{1,2,3\} \times \{1,2\}$ and also on $\{1,2,3\} \times \{1,2,3\}$.

## 2. Problem 2 (2 points)

*This exercise was inspired by N. Vavilov in a summer school in Perugia.* A finite group $G$ has exactly seven conjugacy classes, $C_1 = \{1\}$, $C_2, \ldots, C_7$, such that $C_i^{-1} = C_i$ for all $i = 1, \ldots, 7$ (in other words in $G$ every element is conjugate to its inverse). The first five rows of the character table of $G$ are

| | $\{1\}$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ |
|---|---|---|---|---|---|---|---|
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ |
| $\chi_3$ | 4 | 1 | $-1$ | 0 | 2 | $-1$ | 0 |
| $\chi_4$ | 4 | 1 | $-1$ | 0 | $-2$ | 1 | 0 |
| $\chi_5$ | 5 | $-1$ | 0 | 1 | 1 | 1 | $-1$ |

Calculate the orders of the conjugacy classes of $G$ and complete the character table.

## 3. Problem 3 (1 point)

For each of the following lists $d_1, \ldots, d_k$, answer the question: is there a group $G$ whose complex irreducible character degrees are precisely $d_1, \ldots, d_k$? If not, prove it. If yes, display such a group and prove that its irreducible character degrees are $d_1, \ldots, d_k$.

(1) $1, 1, 2, 2, 2, 2, 2, 2, 2$.
(2) $1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3$.
(3) $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.
(4) $1, 1, 5, 5, 5, 5, 9, 9, 10, 10, 16, 16$.