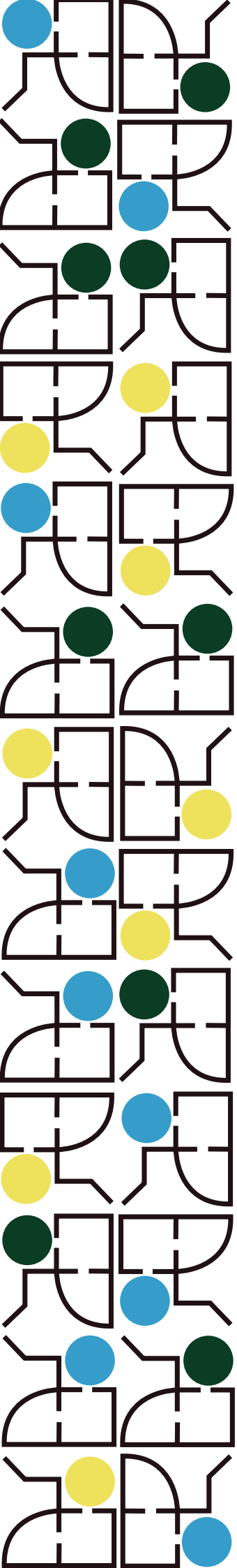

Universidade de Brasília
Departamento de Matemática

XII Summer Workshop in Mathematics

February 10-14, 2020

BOOK OF ABSTRACTS

UnB Instituto de Ciencias Exatas Departamento de Matemática

Universidade de Brasília

XLIX SUMMER SCHOOL

XII SUMMER WORKSHOP IN MATHEMATICS

Book of Abstracts

Universidade de Brasília

Brasília, February of 2020.

Universidade de Brasília

XLIX Summer School
XII Summer Workshop in Mathematics

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Brasília, February of 2020.

Preface

Dear Participants,

It is with great pleasure that we welcome you to the XII Summer Workshop in Mathematics, held between the 10th and 14nd of February 2020, in parallel with the courses of the XLIX Summer School of the Department of Mathematics of the University of Brasilia.

The Summer School was idealized in the early 1970s and, in these more than 40 years of tradition, has promoted the scientific and cultural exchange among its participants in multiple perspectives. These academic interactions are central to the advancement of knowledge and to provide invaluable research collaborations. In fact, the Postgraduate Program in Mathematics at the University of Brasília, currently rated grade 7 at CAPES, has benefited greatly from an active and productive academic environment, and events such as the Workshop were important for our program to achieve this level of excellence.

This event will promote lectures for scientific dissemination and minicourses in different areas of interest, providing Summer School participants with multiple perspectives concerning several mathematical research areas, especially those areas of interest of the researchers at MAT / UnB. The main objective of these activities is the exchange and dissemination of the work developed by the researchers and postgraduate students taking part in the event. Hence, we would like to thank all the support and commitment of the sub-coordinators of the Workshop and of the undergraduate and postgraduate secretary of the Department of Mathematics.

We particularly appreciate the substantial support from the University of Brasília, the FAP-DF and CAPES, which provided us with essential resources for the organization of this event.

Finally, we would like to wish you a great stay in Brasilia and at the University of Brasília. We hope you attend many lectures, interact with several people, learn lots of math and, above all, have fun!

With best regards,

Cristina Acciarri (UnB), Leandro Cioletti (UnB) and Paulo Ruffino (Unicamp)
Coordinators of the XII Summer Workshop in Mathematics and the XLIX Summer School of MAT-UnB

Contents

Plenary Sessions

The Hasse-Witt invariant of generalized Fermat Curves

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Abstract. (Joint work with C. Gonçalves)

For any irreducible algebraic curve $\mathcal{X}$ defined over a field $\mathbb{K}$, its genus $g(\mathcal{X})$ is certainly the most famous birational invariant. If $\mathbb{K}$ has characteristic $p > 0$, then the curve $\mathcal{X}$ has another important birational invariant, called Hasse-Witt invariant or the p-rank of $\mathcal{X}$, which can be defined as the integer $\gamma := \gamma(\mathcal{X}) \in \{0, 1, \ldots, g(\mathcal{X})\}$ for which

$$Pic_0\mathbb{K}(\mathcal{X}, p) = (\mathbb{Z}_p)^\gamma.$$

It is well known that number $\gamma(\mathcal{X})$ is closely related to certain arithmetic and geometric properties of $\mathcal{X}$. For instance, $\mathbb{F}_{p^{2n}}$-maximal and $\mathbb{F}_{p^{2n}}$-minimal curves $\mathcal{X}$ have $\gamma(\mathcal{X}) = 0$, and curves with large p-rank have a somewhat small automorphism group. In this talk, we discuss recent results on the computation of the p-rank of generalized Fermat curves

$$\mathcal{F}_{m,n} : y^m + x^n + 1 = 0,$$

and some of its consequences. In particular, via Deuring-Shafarevich formula, we will present the p-rank of some other important classes of curves such as the Dickson-Guralnick-Zieve curves.

On some graphs of finite groups

Silvio Dolfi (silvio.dolfi@unifi.it)
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Abstract. There are several relevant sets of numerical invariants related to a finite group G: important examples are the sets $\omega(G)$ of the orders of the elements of G, $\mathrm{cs}(G)$ of the sizes of the conjugacy classes of G and $\mathrm{cd}(G)$ of the dimensions of the irreducible complex representations of G.

Given set X of positive integers, one gets an undirected graph $\Delta(X)$ by taking as vertex set the set of prime divisors of the elements in X and by joining two distinct vertices p and q by an edge if there is a number in X which is divisible by the product pq.

We will discuss properties of the graphs that arise by taking as X one of the sets $\omega(G)$, $\mathrm{cs}(G)$ and $\mathrm{cd}(G)$, respectively, and the interplay between properties of the graphs and the algebraic structure of the group G.

An overview on four-manifolds with positive curvature

Ernani Ribeiro Jr (ernanijr@mat.ufc.br)
UFC

Abstract. A classical topic in Geometry is to classify 4-dimensional manifolds with positive sectional curvature. Very few topological obstructions to positive sectional curvature are known, and many conjectures about this subject remain open. In this talk, we will present some open problems related to the classification of 4-dimensional manifolds with positive sectional curvature. Moreover, we will show some partial answers obtained by different approaches.

Bifurcations in Dynamical Systems: from classical towards random

Paulo Ruffino (ruffino@unicamp.br)
University of Campinas, Brazil

Abstract. We start with the observation that most bifurcation in the classical case is related with a topological break in the invariant measures of the system. With simple examples one can see the interesting phenomenon that in random systems – stochastic flows for instance, many of these 'breakings in the topology' can only be verified if one observes two or more, say n, simultaneous trajectories. This is the so called n-point motion in M^n, if M is the state space. A Markov chain with values in a finite space $M = \{1, \ldots, m\}$, $m \geq 2$, has many different extensions to a compatible n-point Markov chain in M^n, for all $1 < n \leq m$. Embedding this phenomenon into the context of stochastic (Lévy) flows of diffeomorphisms in Euclidean spaces, we introduce the notion of an n-point bifurcation of a stochastic flow. Roughly speaking an n-point bifurcation takes place, when a small perturbation of the stochastic flow does not change the characteristics at lower level k-point motions, $k < n$, but does change at the level of n-point motion. We illustrate this phenomenon with an example of an n-point bifurcation, with $n \geq 3$. In addition, we present an algorithm for the detection of the precise level of an n-point bifurcation and a combinatorial formula for the dimension of the vector space of compatible extensions for flows of mappings on M. This is a joint work with Michael Högele.

Gradient elliptic systems with cooperative or competitive interactions: existence, asymptotics and qualitative properties

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Abstract. In this talk we will deal with systems of stationary reaction-diffusion equations appearing in Nonlinear Optics and Bose-Einstein Condensation, where the interaction between different components is either cooperative or competitive. Our aim will be to explain some of the relevant questions that can be asked for each type of interaction, as well as the motivations for its study. We will survey some of the results proved in the last few years, discussing in general the existence and characterization of positive solutions, together with some concentration results. Furthermore, we will explain how a strong competition induces a phase separation phenomenon and gives rise to a free boundary problem. In the last part of the talk, we will consider nonlocal interaction terms between the components. We will highlight some of the similarities and differences between the local and the nonlocal cases, showing some recent results in the nonlocal one.

Lesson Study as a Paradidactic Infrastructure for Development of Mathematics Teacher Knowledge

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Abstract. Lesson Study is a format for teachers work outside the classroom which originated in Japan, in which a group of teachers, often guided by a university researcher, designs and experiments a "research lesson" in view of gaining knowledge about the teaching of a specific mathematical subject, as well as on more general aspects of teaching. It has since been transposed to other countries. In my talk, after a general introduction to the nature and characteristics of this format, I will report on results from research on cases from Japan, Denmark and Brazil, drawing on joint works with T. Miyakawa, J. Bahn, P. Baltar and A. Dias, respectively.

Algebra

Talks

The pro-$\mathcal{C}$ completion of the fundamental group of infinite graphs of groups

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Abstract. The profinite version of the Bass-Serre theory can be used very effectively in the study of certain abstract groups. One sees these groups as the fundamental groups of graphs of groups and through this view it is possible to apply geometric techniques to obtain algebraic results. Let $\mathcal{C}$ be an extension closed variety of finite groups, R an abstract group which is residually $\mathcal{C}$ free-by-$\mathcal{C}$ in its pro-$\mathcal{C}$ topology and denote the pro-$\mathcal{C}$ completion of R by $R_{\hat{\mathcal{C}}}$; by a result of Scott (cf. [3]), R is the fundamental group of a graph of finite groups $(\mathcal{G}, \Gamma)$. Define $\widehat{\mathcal{G}}(m)$ $(m \in \Gamma)$ to be the completion of $\mathcal{G}(m)$ with respect to this topology and take Γ as a finite graph; then it is already profinite and we define the graph of groups $(\widehat{\mathcal{G}}, \Gamma)$ in a natural way. Luis Ribes and Pavel Zalesski showed in [1] that $(\widehat{\mathcal{G}}, \Gamma)$ is injective (i.e., the restriction of the map $\nu : \widehat{\mathcal{G}} \to \Pi_1^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ to each fiber $\mathcal{G}(m)$ $(m \in \Gamma)$ is injective), the fundamental pro-$\mathcal{C}$ group $\Pi_1^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ of $(\widehat{\mathcal{G}}, \Gamma)$ is $R_{\hat{\mathcal{C}}}$ and if we assume that $\Pi_1^{abs}(m)$ is closed in the pro-$\mathcal{C}$ topology of $R = \Pi_1^{abs}(\mathcal{G}, \Gamma)$, for every $m \in \Gamma$, the standard tree $S^{abs} = S(\mathcal{G}, \Gamma)$ is canonically embedded in the $\mathcal{C}$-standard $\mathcal{C}$-tree $S = S^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ with S^{abs} dense in S. We generalise these results to the case when Γ is an infinite graph, answering Open Question 6.7.1 of [2]. It is much more subtle than the previous case, because Γ is not automatically a profinite graph. This is a joint work with Pavel A. Zalesski and it is supported by CAPES.

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On minimal covering of a finite nilpotent group by proper normalizers

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Abstract. A *normalizer covering* of G is a covering Γ of G such that all its members are the normalizers of some non-normal subgroups of G. We determine the structure of finite nilpotent groups without a normalizer covering and we determine the size of a minimal normalizer covering of a finite nilpotent group admitting with a normalizer covering.

Non-abelian tensor square of groups: a commutator approach

Raimundo de Araújo Bastos Jr (bastos@mat.unb.br)
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Abstract. Let G be a group. We denote by $\nu(G)$ a certain extension of the non-abelian tensor square $G \otimes G$ by $G \times G$. This construction is due to N. R. Rocco [4]. In the talk we will discuss some recent progress and questions concerning these constructions [1, 2, 3]. The main idea involves the study of structural results of the non-abelian tensor square (and related construction) in terms of the commutators $\Gamma(\nu(G)) = \{[\alpha, \beta] \mid \alpha, \beta \in \nu(G)\}$.

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Primeness property for graded central polynomials of verbally prime algebras

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Abstract. Let F be an infinite field. The primeness property for central polynomials of $M_n(F)$ was established by A. Regev, i.e., if the product of two polynomials in distinct variables is central, then each factor is also central. In this work we consider the analogous properties for $M_n(F)$ and determine, within the elementary gradings with commutative neutral component, the ones that satisfy this property, namely the crossed product gradings. Next, we consider $M_n(R)$, where R admits a regular grading, with a grading such that $M_n(F)$ is a homogeneous subalgebra and provide sufficient conditions - satisfied by $M_n(E)$ with the trivial grading - to prove that $M_n(R)$ has the primeness property if $M_n(F)$ does. We also prove that the algebras $M_{a,b}(E)$ satisfy this property for ordinary central polynomials. Hence we conclude that, over a field of characteristic zero, every verbally prime algebra has the primeness property. This is a joint work with Diogo Diniz.

Hopf-Galois extensions, regular subgroups of holomorphs, and skew braces

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Abstract. The study of Hopf-Galois extensions can be translated into the study of the regular subgroups of a certain permutational holomorph, which is in turn equivalent to the study of skew

braces. We will be reporting on recent joint work on the subject, done together with Elena Campedel and Ilaria Del Corso.

Conciseness and strong conciseness in profinite groups

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Abstract. A group word w is said to be concise in a class $\mathcal{C}$ of groups if, for each G in $\mathcal{C}$ such that G_w is finite, also $w(G)$ is finite. For topological groups, especially profinite groups, a variation of the classical notion arises quite naturally: we say that w is strongly concise in a class $\mathcal{C}$ of topological groups if, for each G in $\mathcal{C}$, already the bound $|G_w| < 2^{\aleph_0}$ implies that $w(G)$ is finite.

In this talk we discuss conciseness of some words in the class of all residually finite groups and strong conciseness in the class of all profinite groups. We will show that multilinear commutator words are strongly concise in the class of all profinite groups and that every group word is strongly concise in the class of nilpotent profinite groups. From this we can deduce, for instance, that, if w is one of the group words x^2, x^3, x^6, $[x^3, y]$ or $[x, y, y]$, then w is strongly concise in the class of all profinite groups.

The q-Tensor Square of a Powerful p-Group, $q \geq 0$

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Abstract. Let p be a prime number and G a finite p-group. We say that G is powerful if $[G, G] \leqslant G^p$, for p odd, or if $[G, G] \leqslant G^4$, for $p = 2$. If N is a normal subgroup of G and satisfies $[N, G] \leqslant N^p$, for $p \geq 3$, or $[N, G] \leqslant N^4$, for $p = 2$, then N is said to be powerfully embedded em G. In this talk we consider the group $\nu^q(G)$, q a non-negative integer, as described for instance by Bueno and Rocco in [1], which happens to be an extension of the q-tensor square $G \otimes^q G$ by $G \times G$. Our purpose is to address some results concerning $\nu^q(G)$ and $G \otimes^q G$, under the assumption that G is a powerful p-group, which generalize results for $q = 0$ due to Moravec. More specifically:

Theorem A. *Let G be a powerful, finite $p-$group and q a non-negative integer. Then*

(i) $\gamma_i(\nu^q(G))$ is powerfully embedded in $\nu^q(G)$, for $i \geq 2$;

(ii) $\nu^q(G)_i$ is powerfully embedded in $\nu^q(G)$, for $i \geq 1$.

Here, as usual $\gamma_i(\nu^q(G))$ (resp., $\nu^q(G)_i$) denotes the i-th term of the lower central series (resp., derived series) of $\nu^q(G)$.

Let $d(G)$ (resp., $\exp(G)$) denote the minimal number of generators (resp., the exponent) of the group G.

Theorem B. *Under the hypothesis of Theorem A, assume that* $\exp(G)$ *divides q and write d for $d(G)$. Then*

(i) $G \otimes^q G$ is powerfully embedded in $\nu^q(G)$;

(ii) $d(G \otimes^q G) \leq d(d+1)$.

In contrast with the case $q = 0$, for $q \geq 1$ the q-tensor square $G \otimes^q G$ involves a certain subgroup $K \trianglelefteq \nu^q(G)$, which plays an important role in its structure.

Theorem C. *Let G be a powerful p-group. Then*

(i) $\exp([\nu^q(G), \nu^q(G)])$ *divides* $\exp(G)$*;*

(ii) $\exp(K)$ *divides* $\exp(G)$ *if p is odd or if* $4 \mid q$*;*

(iii) $\exp(K)$ *divides* $2\exp(G)$ *if* $p = 2$ *and* $4 \nmid q$*.*

* This is a joint work with Noraí Romeu Rocco.

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Finitely presented restricted metabelian Lie algebras over perfect fields

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Abstract. Lie algebras over fields of characteristic $p > 0$ often have an additional structure involving a special class of applications of given algebra in itself and whose properties related the elevate to the p-th power and the Lie bracket. Such a structure was first studied by Jacobson in [6] and called by him *restricted Lie algebras*. Intuitively, we can consider a restricted Lie algebra as an algebra with a double structure, the first one, the usual Lie algebra given by its product and, a second one, the restricted structure given by the new operation defined in it. The restricted Lie algebras and their homological properties are our object of study.

Little is known about homological properties of the restricted Lie algebras and its finite presentability. The main objective of our research is to determine a criterion to study when a restricted metabelian Lie algebra is finitely presented and when is the homological type FP_2. Using this restricted homology techniques and the results obtained by Bryant and Groves (see [1] and [2]) for the case of the metabelian Lie algebra we give an answer to the problem in the specific case that the Lie algebra is defined over a perfect field of positive characteristic.

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On locally nilpotent groups having proper contranormal subgroups

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Abstract. A subgroup H of a group G is **contranormal in** G if $H^G = G$, where $H^G = \langle x^{-1}hx \mid h \in H,\ x \in G\rangle$ is the normal closure of H in G. For example G is contranormal in G. Moreover, every subgroup of a finite group G is a contranormal subgroup of a subnormal subgroup of G. The concept of contranormal subgroup has been introduced by J.S. Rose in the paper [3]. Contranormal subgroups have been studied for example in the paper [2]. Obviously a contranormal subgroup H of a group G is normal or subnormal in G if and only if $H = G$. It follows that groups whose subgroups are all subnormal, in particular nilpotent groups, do not contain proper contranormal subgroups. The converse is also true for finite groups.

The aim of this talk is to present some results obtained in the locally nilpotent case in the paper [1]. We first notice that locally nilpotent groups can have proper contranormal subgroups. We describe locally nilpotent groups having a proper finite contranormal subgroup. We also investigated the structure of a locally nilpotent group having either a finitely generated or a Cernikov proper contranormal subgroup.

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Color Involutions on Graded Associative Algebras

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Abstract. Let G an abelian group and F a field. In 2016 K. Sousa and I. Sviridova characterized primitive G-graded associative rings (F-algebras) with a minimal G-graded right ideal and a σ-involution in terms of nondegerate ε-hermitian sesquilinear graded forms, when G is a cyclic group of prime order and σ an anti-symmetric 2-cocycle. Also, there they left the open question: Is it possible to obtain this characterization when G is an abitrary finite abelian group?

In this talk we will present a positive answer to the above question when G is an arbitrary abelian group and σ an anti-symmetric bicharacter. This, as well as the characterization of Sousa and Sviridova, generalizes some results of Racine for superinvolutions [2], and of Bahturin, Bresăr and Kochetov for graded involutions [1]. This is a joint work with I. Sviridova.

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A characterization of permutation modules extending a theorem of Weiss

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Abstract. When G is a finite p-group, a famous theorem of A. Weiss gives a sufficient (but not necessary) condition for a $\mathbb{Z}_pG$-lattice to be a permutation module, looking only at its restriction to a normal subgroup N and its N-fixed points. In case N has order p, I'll give a variant of Weiss' Theorem that characterizes $\mathbb{Z}_pG$-permutation modules. Joint work with Pavel Zalesskii.

A polycyclic presentation for the q-tensor square of the dihedral group D_{2n}, q odd

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Abstract. Let G be a group and q a non-negative odd integer. We denote by $E_q(G)$ a q-central extension of G by a finite abelian group T. In this work we determine the q-tensor square of the dihedral group of order $2n$ for q odd , via its embedding into $E_q(G)$.

Non-metacyclic groups acting with nilpotent centralizers

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Abstract. Let A be a non-metacyclic finite group. Suppose that A acts coprimely on a finite group G in such a manner that $C_G(a)$ is nilpotent for any $a \in A^{\#}$. In this talk, we investigate some

conditions on A which imply that G is nilpotent with "bounded" nilpotency class. In particular, we generalize known results on action of q-groups and Frobenius groups.

Monomial identities and almost non-degenerate gradings on matrix algebras

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Abstract. Let G be a group. A G-grading on the matrix algebra $R = M_n(F)$ is called elementary, if each matrix unit e_{ij} is homogeneous in this grading. There are many results in the literature concerning the graded identities of R. In particular, if the neutral component is the set of diagonal matrices, then the graded identities of R follow from three basic types of identities and monomial identities of length ≥ 2 bounded by a function $f(n)$.

In this talk we will present a recent and complete result about the optimality of the bound function $f(n)$.

When G is a linearly ordered abelian group, we will show necessary conditions so that a G-grading on R is almost non-degenerate, i.e., a grading in which the monomial identities are consequences of monomial identities of length one.

For the case $G = \mathbb{Z}$, and $n \leq 5$, we give a complete characterization of all almost non-degenerate gradings on R.

These results were obtained in a joint work with Diogo Diniz and Lucio Centrone.

On groups with some subgroups complemented

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Abstract. It is well-known that if a group has a family of subgroups satisfying the same property then one can grasp several information about the structure of the whole group (see for example [3, 4]). For instance, in a group of infinite rank the family of all subgroups of infinite rank has a strong influence on the group itself as showed in [1] and [2]. Recall that a group G has finite rank $\rho(G) = r$ if every finitely generated subgroup of G can be generated by at most r elements and r is the least positive integer with this property. If such an integer r does not exist, then we say that G has infinite rank.

The property we are mainly interested in is the complementation. We say that a subgroup H of a group G is complemented in G if there exists a subgroup K of G such that $G = HK$ and $H \cap K = 1$. The subgroup K is called a complement of H in G. In [5] Hall proved that a finite group has every subgroup complemented if and only if it is supersoluble with elementary abelian Sylow subgroups.

The aim of this talk is to deal with some classes of infinite groups. Indeed, we will characterize two families of infinite groups in which some classes of subgroups are complemented, facing in particular the case of infinite rank groups.

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Exponent of a finite group admitting a coprime automorphism

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Abstract. Let G be a finite group admitting a coprime automorphism ϕ of order n. Denote by G_ϕ the centralizer of ϕ in G and by $G_{-\phi}$ the set $\{x^{-1}x^\phi;\ x \in G\}$. In this talk, I will present some results bounding the exponent of G. In particular, I will give an idea of the proof of the following theorem:

Suppose that G_ϕ is nilpotent of class c. If $x^e = 1$ for each $x \in G_{-\phi}$ and any two elements of $G_{-\phi}$ are contained in a ϕ-invariant soluble subgroup of derived length d, then the exponent of $[G, \phi]$ is bounded in terms of c, d, e, n.

This is a joint work with Pavel Shumyatsky.

Lie nilpotence and solvability of Poisson algebras

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Abstract. We investigate the conditions for an arbitrary Poisson algebra to be Lie nilpotent or solvable. The special case of (truncated) symmetric Poisson algebras is also discussed.

Hook Theorem for PI-Superalgebras with Superinvolution

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Abstract. Let $A = A_0 \oplus A_1$ be a superalgebra over a field F of characteristic zero. A superinvolution in A is a graded linear application $\# : A \to A$ such that $(c^\#)^\# = c$ for all $c \in A$ and $(ab)^\# = (-1)^{deg(a)deg(b)} b^\# a^\#$ for all homogeneous elements $a, b \in A_0 \cup A_1$, where $deg(d)$ is the homogeneous degree of $d \in A_0 \cup A_1$. In this case, we say that A is a $\#$-superalgebra.

The study of superalgebras with some superinvolution and their identities is of the great interest for several areas of Mathematics.

One of the important results of the theory of polynomial identities is the celebrated Hook Theorem, which was proven by Amitsur and Regev in [1]. There exist also versions of this theorem for the case of $\mathbb{Z}_2$-graded identities and identities with involution that were proved by Regev and Giambruno in [2]. These results are related with the using of theory of group representations for the

understanding of a behaviour of identities and, moreover, they have various applications in PI-theory and in other areas of Mathematics.

The main goal of this talk is to present a version of the Hook Theorem for the case of superidentities with superinvolution.

For a superalgebra with superinvolution over a field F of characteristic zero, the ideal of superidentities with superinvolution is completely defined by multilinear identities that have a structure of $S_{\langle n\rangle}$-modulo, where $\langle n\rangle = (n_1, n_2, n_3, n_4)$, $n = n_1 + n_2 + n_3 + n_4$, and each n_i corresponds to the quantity of homogeneous #-symmetric or #-antisymmetric variables. The behavior of these #-superidentities may be described by the corresponding cocharacter

$$\chi_{\langle n\rangle}(A) = \sum_{\langle\lambda\rangle\vdash\langle n\rangle} m_{\langle\lambda\rangle}\chi_{\langle\lambda\rangle},$$

where $\langle\lambda\rangle = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ is a multipartition of $\langle n\rangle$ and $\lambda_i \vdash n_i$ is a partition of n_i.
Theorem: *(Hook Theorem for #-superalgebras)* Let A be a #-superalgebra. If A is a PI-algebra (i.e., A also satisfies some non-trivial ordinary identity), then there exist integers $d_i, l_i \geq 0$, with $i \in \{1, 2, 3, 4\}$, such that the n-th cocharacter, $\chi_{\langle n\rangle}(A)$, is contained in a quadruple hook

$$H_4(n) = (H(d_1, l_1), H(d_2, l_2), H(d_3, l_3), H(d_4, l_4)),$$

that is,

$$\chi_{\langle n\rangle}(A) = \sum_{\substack{\langle\lambda\rangle\vdash n \\ \langle\lambda\rangle\in H_4(n)}} m_{\langle\lambda\rangle}\chi_{\langle\lambda\rangle},$$

where $\langle\lambda\rangle \in H_4(n)$ means $\lambda_i \in H(d_i, l_i)$, for all $i \in \{1, 2, 3, 4\}$, that is, $\lambda_{d_{i+1}} \leq l_i$.

This is a joint work with I. Sviridova.

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On the factorability of $T_{C_{p^n}}$-ideals of C_{p^n}-graded upper block triangular matrix algebras

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Abstract. Let F be a field of characteristic zero. In 2003, Giambruno and Zaicev proved that the T-ideal of any upper block triangular matrix algebra $UT(d_1, \ldots, d_m)$ satisfies the factoring property, that is, $\mathrm{Id}(UT(d_1, \ldots, d_m)) = \mathrm{Id}(M_{d_1}(F)) \cdots \mathrm{Id}(M_{d_m}(F))$.

If F is an algebraically closed field and $G := C_k$ is a cyclic group of order k, consider a G-graded upper block triangular matrix algebra $UT_G(A_1, \ldots, A_m)$ equipped with an elementary G-grading, where $A_1, \ldots, A_m$ are finite dimensional G-simple F-algebras. In 2019, in a joint work with Avelar and Di Vincenzo, we have stated that the T_{C_2}-ideal of $UT_{C_2}(A_1, \ldots, A_m)$ is factorable if, and only if,

there exists at most one index $i \in [1, m]$ such that A_i is a non-C_2-regular simple superalgebra, if and only if, $UT_{C_2}(A_1, \ldots, A_m)$ has only one class of C_2-graded isomorphism.

Recently, in a joint work with Di Vincenzo and Pinto, we have stated a generalization of such results for the T_G-ideals of the upper block triangular matrix algebras $UT_G(A_1, \ldots, A_m)$, in case G is a finite cyclic p-group with p being a prime number.

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Identities for the special linear Lie algebra with the Pauli and Cartan gradings

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Abstract.

Let $\mathbb{K}$ be a field, the graded identities for the algebra $sl_2(\mathbb{K})$ with the Cartan and Pauli gradings and a $\mathbb{Z}_2$-grading that is a coarsening of the Cartan grading were studied in [2], [3], for a field $\mathbb{K}$ of characteristic 0, and in [4], for an infnite field $\mathbb{K}$ with char $\mathbb{K} \neq 2$. In this talk we discuss the graded identities of special linear Lie algebras, over a field of characteristic zero, with the Pauli and Cartan gradings. Given a prime number p we provide a finite basis for the graded identities of $sl_p(\mathbb{K})$ with the Pauli grading by the group $\mathbb{Z}_p \times \mathbb{Z}_p$, as a by-product we determine a basis for the identities of certain graded Lie algebras with a grading in which every homogeneous has dimension ≤ 1. W e also prove that $\mathrm{var}^{\mathbb{Z}_p \ times \mathbb{Z}_p}(sl_p(\mathbb{K}))$ is a minimal variety and satisfies the Specht property . For $sl_m(\mathbb{K})$ with the Cartan grading a finite basis for the graded identities is determined. This is joint work with F . L. de Souza and C. Fidelis in [1].

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Polynomial identities for Leibniz Algebras

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Abstract. A Leibniz algebra over a field is a non-associative algebra with a product satisfying the Leibniz identity $(xy)z = (xz)y + x(yz)$. Leibniz algebras (sometimes called a Loday algebra) can be seen as a generalization non-anticommutative of Lie algebras. Leibniz algebras appear to be

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related in a natural way to several topics such as differential geometry, homological algebra, algebraic K-theory etc. In this talk we speak about polynomial identities for concrete Leibniz algebras of low dimension.

Powerfully nilpotent groups

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Abstract. In this talk we discuss a special subclass of powerful groups called powerfully nilpotent groups. These are finite p-groups that possess a central series of a special kind. We will describe some structure theory and a "classification" in terms of an ancestry tree and powerful coclass.

IBN-varieties of algebras

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Abstract. We consider in a class of all universal algebras of a signature Ω some variety Θ. We denote by $F_\Theta(X)$ the free algebra of this variety generated by the set of free generators X. We consider only finitely generated free algebras. We denote by $|X|$ the quantity of the elements of the set X.

Definition 1. We say that the variety Θ is an **IBN-variety** (or variety which has an **IBN propriety**), if from $F_\Theta(X) \cong F_\Theta(Y)$ we can conclude that $|X| = |Y|$.

The concept of variety with IBN (invariant basic number) propriety first appeared in ring theory. It is known that if we consider some field k, the vector space V over this field such that $\dim_k V = \aleph_0$ ring A of all linear operators on this vector space ($A = \mathrm{End}_k V$), then $A \cong A \oplus A \cong A \oplus A \oplus A \cong \ldots \cong \underbrace{A \oplus \ldots \oplus A}_{n\ times} \cong \ldots$, i.e., the variety ${}_A\mathfrak{M}$ of all rights modules over the ring A has not the IBN propriety.

The proving of the IBN propriety of some variety is very important in universal algebraic geometry. This is a milestone in the study of the relation between geometric and automorphic equivalences of algebras of this variety.

We will look at some examples where the IBN property of certain varieties can be proved directly. For example for every signature Ω the variety defined by the empty set of identities has the IBN property.

We will discuss very simple but very useful

Theorem 1. *If Δ, Θ some varieties of universal algebras of a signature Ω, $\Delta \subset \Theta$ and Δ is an IBN-variety then Θ is also an IBN-variety.*

We will consider applications of this theorem.

We will consider many-sorted universal algebras as well as one-sorted. So all concepts and all results will by generalized for the many-sorted case.

Varieties of algebras with graded involution

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Abstract. In this talk, we will present some results recently obtained about varieties generated by algebras with graded involution.

Profinite groups with abelian centralizers

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Abstract. Profinite groups in which the centralizer of any non-identity element is abelian (i.e., profinite CA-groups) are also known as profinite commutativity-transitivity groups. In this talk I shall present a dichotomy theorem obtained with P. Shumyatsky and P. Zalesskii (2019, Israel J. Math, v. 230): *Any profinite CA-group has a finite index closed subgroup that is either abelian or pro-p.*

We acknowledge financial support from CNPq and FAPDF.

Minicourses

Automorphism groups of rooted trees

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Abstract. In this course we will first introduce the basic theory of the groups of automorphisms of rooted trees and their subgroups. Then we will focus on fractal groups, on (weakly) branch groups and on the congruence subgroup property.

Covering spaces and covering groups

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Abstract. Homology of groups is where representation theory encounter algebraic topology. With this mini–course we offer a panoramic view on covering spaces and covering groups, which provides an instance of the elegant interaction between geometry and algebra. Thus, we focus on algebraic topology first, recalling several interesting facts about covering spaces, the homotopy and homology groups, the theorems of Hurewicz, the Eilenberg - Maclane complexes and, finally, the Hopf formula. Then we move to the viewpoint of abstract algebra, with emphasis on Schur's theory on projective representation. We recall the motivating problem in representation theory, and so we encounter the Schur multiplier and few elements of the theory of group extensions. In conclusion, we gather the above material to see how covering groups carry beautiful geometries into group homology.

Posters

Gradings and Involutions on Matrix Algebras

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Abstract. We will consider $\mathbb{F}$ an algebraically closed field of characteristic zero, G an Abelian group, and $\mathcal{A} := M_n(\mathbb{F})$ a Matrix algebra over $\mathbb{F}$ graded by G. In this presentation we are going to describe graded involutions on $\mathcal{A}$. We will show how this description is defined by a non-degenerate bilinear form and the two graded subspaces $K(\mathcal{A}, *)$ and $H(\mathcal{A}, *)$, respectively a Lie subalgebra and Jordan subalgebra, both arising due to the involution. The presentation is based on the results published in [1]-[4].

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Automorphisms of the category of finitely generated free groups of the some subvariety of variety of groups

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Abstract.

Let Θ be a variety of universal algebras. We consider the category Θ^0. The objects of this category are finitely generated free algebras of the variety Θ. The morphisms of this category are homomorphisms of these algebras. Let $\mathfrak{A}$ be the group of all automorphisms of Θ^0 and let $\mathfrak{Y}$ be the group of all inner automorhisms of Θ^0. On the previous papers ([1] and [2]) it was proved that when Θ is the variety of all groups, or all abelian groups, or all nilpotent groups with nilpotency class $d \geq 2$, the quotient group $\mathfrak{A}/\mathfrak{Y}$ is trivial. It means that the automorphic equivalence of groups coincides with the geometric equivalence.

We represent the first example of a subvariety of the variety of all groups such that the group $\mathfrak{A}/\mathfrak{Y}$ has 2 elements. In this subvariety it is possible to exist two groups that are automorphically equivalent but are not geometrically equivalent. The work was done under the supervision of professor A. Tsurkov.

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An infinite-dimensional abelian ideal in the weak commutativity construction

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Abstract. We will present how we can show that the abelian ideal $R(\mathfrak{g}) \subset \chi(\mathfrak{g})$ in the weak commutativity construction for Lie algebras [1] may be infinite-dimensional. In particular, $\chi(\mathfrak{g})$ has infinite cohomological dimension if $\mathfrak{g}$ is free non-abelian. This is done via methods of Gröbner-Shirshov bases.

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Polynomial Identities of the Weyl Algebra

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Abstract.

Let $\mathbb{F}$ be a field and X an enumerable set. The *free associative algebra* of X over $\mathbb{F}$ is the algebra $\mathbb{F}\langle X\rangle$ of polynomials on the non-commutative variables $x \in X$. Let A be an $\mathbb{F}$-algebra and $f = f(x_1, \ldots, x_n) \in \mathbb{F}\langle X\rangle$. We say that $f \equiv 0$ is a *polynomial identity of* A if $f(a_1, \ldots, a_n) = 0$ for all $a_1, \ldots, a_n \in A$. It is obvious that the trivial polynomial is an identity for any algebra. Than, if A satisfies a non-trivial identity $f \equiv 0$, we say that A is a *PI-algebra*.

We are interested in studying polynomial identities for the Weyl algebra A_n. Define the algebra A_1 as the non-commutative algebra over $\mathbb{F}$ generated by x and y with the relation $yx = xy + 1$, i. e., $A_1 = \dfrac{\mathbb{F}\langle x, y\rangle}{(yx - xy - 1)}$. For $n > 1$, we define the n-th Weyl algebra recursively as $A_n = A(A_{n-1})$. It is a known fact that if the field $\mathbb{F}$ has characteristic zero, then the Weyl algebra admits no nontrivial identities (see [3]). However, in case $\mathbb{F}$ has characteristic $p > 0$, there do exist identities for the Weyl algebra. For example, the standard polynomial St_4 (respec. St_6) is an indentity of A_1 in case $p = 2$ (resp. $p = 3$), being

$$St_m(x_1, \ldots, x_m) = \sum_{\sigma \in Sm} sgn(\sigma) x_{\sigma(1)} \ldots x_{\sigma(m)}$$

the standard polynomial of degree m. In this work, we study polynomial identities in the case where the characteristic of the field is positive.

This work was done under the supervision of professor Artem Lopatin, from Universidade Estadual de Campinas (Unicamp).

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Topics in Generalized Hamming Weights

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Abstract. In Coding Theory, the minimum distance of a code it is not the lone subject of interest. Also, the Hamming weights of higher order are important because this data collection, called the distribution of weights, is closely related to the code performance. Since the definition of generalized Hamming weights given by Wei, many works that investigate these parameters for different classes of codes have been produced. So far, there are only isolated results on the weights of various types of codes. The aim of this work is to systematize the known results regarding the distribution of weights for some types of codes.

Analysis

Talks

Adams' trace principle on Morrey-type spaces over β-Hausdorff dimensional surfaces

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Abstract. In this talk we strengthen to Morrey-Lorentz spaces the Sobolev-trace principle discovered by D. R. Adams and extended to another functions spaces by Adams, Xiao and Liu. More precisely, we show that Riesz potential I_α mapping

$$I_\alpha : \mathcal{M}^{\lambda}_{pl}(\mathbb{R}^n, d\nu) \longrightarrow \mathcal{M}^{\lambda_\star}_{qs}(\Omega,\, d\mu),$$

continuously if and only if the Radon measure μ satisfies $\mu(B_r(x)) \lesssim r^\beta$ for every $x \in \mathrm{supp}(\mu) \subset \Omega$ and $r > 0$, provided $n - \alpha p < \beta \leq n,\ \alpha = \frac{n}{\lambda} - \frac{\beta}{\lambda_*}$ and $\frac{\lambda_*}{q} \leq \frac{\lambda}{p}$. The brand-new is if $\mu(B_r(x)) \lesssim r^{k/2}$ over smooth surfaces Ω with k non-vanishing principal curvatures at each $x \in \mathrm{supp}(\mu)$, then tracing principle applies.

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A global minimization trick to solve some classes of Berestycki-Lions type problems

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Abstract. In this paper we show an abstract theorem that can be used to prove the existence of solution for a class of elliptic equation considered in Berestycki-Lions and related problems. Moreover, we use the abstract theorem to show that a class of zero mass problems has multiple solutions, which is new for this type of problem.

A two-phase free boundary problem ruled by the infinity Laplacian

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Abstract. In this talk we discuss some regularity aspects for solutions of a non-variational two-phase free boundary problem ruled by the infinity Laplacian. This is a joint work with E. Teixeira (University of Central Florida - USA) and J.M. Urbano (Universidade de Coimbra - PT).

Existence and multiplicity of positive solutions for a singular system via sub-supersolution method and Mountain Pass

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Abstract. In this paper we show existence and multiplicity of positive solutions using the sub-supersolution method and Mountain Pass theorem in a general singular system which the operator is not homogeneous neither linear.

Regularity theory for a class of variable-exponent fully nonlinear elliptic equations

Anne Caroline Bronzi (annebronzi@gmail.com)
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Abstract. In this talk we will explore the regularity of viscosity solutions for a class of variable-exponent, degenerate/singular elliptic equations in non-divergence form.

On a splitting of the Nehari manifold via the generalized Rayleigh quotients

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Abstract. In this work we deal with equations that have variational structure and their Nahari manifolds may contain more than two different types of critical points. We propose a method of separating critical points on the Nehari manifold based on using of nonlinear generalized Rayleigh quotients. We illustrate our approach by establishing existence of positive solutions, ground states and multiplicity results for a two-parameter nonlinear elliptic boundary problem with polynomial nonlinearities.

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Stationary Kirchhoff equations involving critical growth and vanishing potential

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Abstract. We establish the existence of positive solutions for a class of stationary Kirchhoff-type equations defined in the whole $\mathbb{R}^3$ involving critical growth in the sense of the Sobolev embedding

and potentials, which may decay to zero at infinity. We use minimax techniques combined with an appropriate truncated argument and a priori estimate. These results are new even for the local case, which corresponds to nonlinear Schrödinger equations.

Elliptic equations with exponential nonlinearity combined with convection term

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Universidade Federal de Juiz de Fora

Abstract. In this talk, we will present an existence result of positive solutions for a nonlinear elliptic Dirichlet problem in dimension N involving the N-Laplacian. The nonlinearity considered depends on the gradient of the unknown function and an exponential term. Our approach is based on the approximation scheme, where we consider a new class of normed spaces of finite dimension.

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On singular elliptic boundary value problems via a harmonic analysis approach

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Abstract. We consider a class of elliptic problems in the half-space $\mathbb{R}^n_+$ with nonhomogeneous boundary conditions containing nonlinearities and critical singular potentials. We obtain existence and regularity results by means of a harmonic analysis approach based on a framework of weighted spaces in Fourier variables. This framework seems to be new in the context of elliptic boundary value problems and allows us to consider Hardy's potential $\lambda_1/|x|^2$ in $\mathbb{R}^n_+$ and Kato's potential $\lambda_2/|x'|$ on the boundary $\partial\mathbb{R}^n_+$, as well as their versions with multiple poles, without using the so-called Kato and Hardy inequalities. Singular boundary forcing terms can also be addressed. Moreover, our results cover supercritical nonlinearities, such as $\pm u^p$ in $\mathbb{R}^n_+$ and $\pm u^q$ on $\partial\mathbb{R}^n_+$ with integers $p > 2^* - 1$ and $q > 2_* - 1$.

Joint work with Nestor F. Castañeda-Centurión (UESC, BR)

AMS MSC: 35J60; 35J66; 35J65; 35J75; 42B35, 42B37

KEYWORDS: Nonlinear elliptic boundary value problems; Harmonic analysis; Singular potentials; Supercritical nonlinearities; Existence; Regularity

On (p, N) problems with critical exponential nonlinearities

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Abstract. In this talk, we discuss about recent results for equations in $\mathbb{R}^N$ driven by (p, N) elliptic operators of Marcellini type and involving critical Trudinger–Moser nonlinearities with exponential growth. Our variational equations present some difficulties due to the double structure of the elliptic part combined with the double lack of compactness at critical level. In order to state a multiplicity result, we introduce a tricky step analysis based on the application of a completely new Brézis and Lieb type lemma for exponential nonlinearities.

In the second part of the talk, we present a nontrivial generalization of the (p, N) equations on a vectorial system, with a coupled exponential nonlinearity. This situation forces more restrictive assumptions on the coupled exponential term than the related scalar counterpart. In particular, we introduce conditions which allow us to prove that both components of the vectorial solutions are nontrivial and different. That is, the system does not reduce into an equation.

All results of the talk are obtained in the papers [1, 2, 3].

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DOI: https://doi.org/10.1016/j.jmaa.2019.123379

Existence, nonexistence and multiplicity of positive solutions for an equation with degenerate nonlocal diffusion

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Abstract. An elliptic boundary value problem with degenerate nonlocal term is considered. The existence, nonexistence and multiplicity of positive solutions for the problem are studied. The results are obtained in two different cases: under monotonicity assumptions on the nonlinearity term as well as without monotonicity conditions. Some order in the set of obtained solutions is provided.

Non-Local Degenerate Diffusion Coefficients Break Down the Components of Positive Solutions

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Abstract. This work deals with nonlinear elliptic problems where the diffusion coefficient is a degenerate non-local term. We show that this degeneration implies the growth of the complexity of

the structure of the set of positive solutions of the equation. Specifically, when the reaction term is of logistic type, the continuum of positive solutions breaks into two disjoint pieces. Our approach uses mainly fixed point arguments.

On the extreme value of the Nehari manifold method for a class of Schrödinger equations with indefinite weight functions

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Abstract. In this lecture we are concerned with the following class of equations

$$-\Delta_p u - \lambda h(x)|u|^{p-2}u = f(x)|u|^{\gamma-2}u, \quad \text{in } \mathbb{R}^N,$$

involving indefinite weight functions. The existence of solution may depend on the parameter λ. We analyze the extreme value λ^* and we study its relation with the Nehari manifold. Our goal is to establish the existence of two solutions when $\lambda > \lambda^*$.

On a class of quasilinear equations involving critical exponent and nonlinearity concave at the origin

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Abstract. In this talk, we are interested in solving two class of quasilinear problems, namely,

$$\left\{ \begin{array}{l} -\Delta u - u\Delta(u^2) = -\lambda|u|^{q-2}u + \mu u + (u^+)^{p-1}, \text{ in } \Omega, \\ u = 0, \text{ on } \partial\Omega, \end{array} \right\}. \tag{P}$$

and

$$\left\{ \begin{array}{l} -\Delta u - u\Delta(u^2) = -\lambda|u|^{q-2}u + |u|^{22^*-2}u + \mu g(x,u) \text{ in } \Omega, \\ u = 0, \text{ on } \partial\Omega, \end{array} \right. \tag{Q}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with regular boundary $\partial\Omega$ and $22^* = 4N/(N-2)$. For problem (P), we assume $N \geq 1$, $\lambda, \mu > 0$, $1 < q < 2$ and $4 < p < 22^*$ if $N \geq 3$ or $p > 4$ if $N = 1$ or $N = 2$ and prove the existence of two nontrivial classical solutions for problem (P) for $\lambda > 0$ fixed and for any $\mu > 0$ large enough. Firstly, we make a correct change of variable $u = f(v)$, introduced in [2], which relates problem (P) with a semilinear problem that possesses two classical solutions obtained via Mountain Pass Theorem. It is also proved that the correspondent minimax levels converge to zero as $\lambda \to 0^+$. Moreover, one of the solutions is nonnegative and the other is nonpositive.

For problem (Q), we assume $N \geq 3$, $\lambda, \mu > 0$, $1 < q < 4$ and g has a subcritical growth and possesses a condition of monotonicity. Introducing a new type of a Nehari set, more specifically, defining

$$\mathcal{N} = \left\{ w \in H_0^1(\Omega) \setminus \{0\};\ I'_{\lambda,\mu}(w)\frac{f(w)}{f'(w)} = 0 \right\},$$

where $I_{\lambda,\mu}$ is the energy functional associated with problem (Q) and f is the change of variable, we note that every critical point of $I_{\lambda,\mu}$ is contained in $\mathcal{N}$ and prove that one of the following cases is valid: 1. Problem (Q) has three solutions, one of which is nodal and ground state solution, one is nonnegative and the other is nonpositive; 2. Problem (Q) has two solutions, one of which is nonnegative and ground state solution and the other is nonpositive; 3. Problem (Q) has two solutions, one of which is nonpositive and ground state solution and the

other is nonnegative, whenever $\lambda > 0$ is fixed and $\mu > 0$ is large enough. With ground state solution, we mean a nontrivial solution $u_0 \in H_0^1(\Omega)$ that satisfies $I_{\lambda,\mu}(u_0) = \inf_{\mathcal{N}} I_{\lambda,\mu}(w)$.

In order to prove the existence result for problem (Q), some difficulties naturally arise. One of them is that we had to pass for a Brezis-Lieb Lemma applied in a term that depends on f, the change of variable. So we studied the growth of the change f, what allowed to use Theorem 2 in [1] (general Brezis-Lieb Lemma) to overcome this difficulty.

This is a joint work with G. M. Figueiredo, R. Ruviaro and E. L. de Moura.

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On a Hardy type inequality and its applications

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Abstract. In this talk we present a new Hardy type inequality and as a consequence we establish embedding results of a certain Sobolev space defined on the upper half-space. We also present some applications of this embeddings results in the study of some class of elliptic problems on the half-space with nonlinear boundary conditions.

On a class of Hamiltonian Choquard-type elliptic systems

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Abstract. This work is concerned with the existence of solutions for a class of Hamiltonian Choquard-type elliptic systems of the form

$$\begin{cases} -\Delta u + V(x)u = \left(\frac{1}{|x|^\mu} * G(v)\right) g(v) \text{ in } \mathbb{R}^2, \\ -\Delta v + V(x)v = \left(\frac{1}{|x|^\mu} * F(u)\right) f(u) \text{ in } \mathbb{R}^2, \end{cases}$$

where $0 < \mu < 2$, V is a potential that may change sign, the nonlinear terms f and g have exponential critical growth, $F(t) = \int_0^t F(\tau)d\tau$, and $G(t) = \int_0^t G(\tau)d\tau$. Typical features of this class of problems are a lack of compactness because of the unboundedness of the domain and that the critical growth and the energy functional associated with this class of systems are strongly indefinite, that is, the domain has a saddle-point geometry where both positive and negative subspaces of the quadratic form are infinite-dimensional. Moreover, the presence of the nonlocal term in the nonlinearity provides a

series of difficulties in obtaining certain estimates. The research was conducted jointly with Bráulio B.V. Maia-UFRA.

Convex topological algebras via linear vector fields and Cuntz algebras

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Abstract. In this talk, we describe realization by linear vector fields for any Lie algebra which admits a biorthogonal system and for its any suitable representation. The embedding into Lie algebras of linear vector fields is in analogue to the classical Jordan-–Schwinger map. A number of examples of such Lie algebras of linear vector fields is computed. In particular, we obtain examples of the twisted Heisenberg-Virasoro Lie algebras and the Schrödinger-Virasoro Lie algebras among others. More generally, we construct an embedding of an arbitrary locally convex topological algebra into the Cuntz algebra. In the end, we give explicit formula of immersion of arbitrary locally convex finite dimensional topological algebra into certain class of dynamical systems.

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Semilinear parabolic equations with asymptotically linear growth

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Abstract. We present some recent work on the existence and behaviour of solutions for a class of semilinear parabolic equations, defined on a bounded smooth n-dimensional domain, and we assume that the nonlinearity is asymptotically linear at infinity. We analyze the behavior of the solutions when the initial data varies in the phase space. We obtain global solutions which may be bounded or blowup in infinite time (grow-up). Our main tools are the comparison principle and variational methods. Particular attention is paid to initial data at high energy level. We use the Nehari manifold to separate the phase space into regions of initial data where uniform boundedness or grow-up behavior of the semiflow may occur. This is based on a joint work with L. Maia.

Existence and multiplicity of positive solutions for a fourth-order elliptic equation

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Abstract. We prove existence and multiplicity of solutions for the problem

$$\begin{cases} \Delta^2 u + \lambda \Delta u = |u|^{2^*-2}u, \text{ in } \Omega, \\ u, -\Delta u > 0, \text{ in } \Omega, \quad u = \Delta u = 0, \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 5$, is a bounded regular domain, $\lambda > 0$ and $2^* = 2N/(N-4)$ is the critical Sobolev exponent for the embedding of $W^{2,2}(\Omega)$ into the Lebesgue spaces.

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On a 3D phase-field model with convection under a magnetic field effect

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Abstract. In this talk we discuss a three-dimensional isothermal model of solidification for a binary alloy with melt convection and under a magnetic field effect. The model consists of a highly nonlinear system of partial differential equations for the state variables: the velocity field, the pressure, the potential function of the electrical field, the phase-field which represents the solid/liquid phase of the alloy, and the concentration. The well-posedness of the model is presented. Moreover, the existence of solutions when the diffusion coefficient of the concentration equation vanishes for some values of the phase-field is investigated.

Topological singularities of Sobolev maps

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Abstract. Sobolev maps $u : M \to N$ between two smooth Riemannian manifolds M and N may inherit topological singularities from both the domain and target. This phenomenon can have analytical consequences as it prevents smooth maps from being close to u with respect to the Sobolev norm. The talk, based on a joint work with P. Bousquet (Toulouse) and J. Van Schaftingen (UCLouvain), will present how such topological singularities arise and how one can detect and quantify them for any given Sobolev map.

Existence and non-existence results of dead cores for fully nonlinear elliptic problems

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Abstract. In this talk we going to show existence/non-existence results of positive solutions for second order equations as follows

$$(\mathrm{P}_{a,q}) \qquad \begin{cases} |Du|^{\gamma}F(x,D^2u)+a(x)u^q(x) &= 0 \quad \text{in} \quad \Omega, \\ u &\geq 0 \quad \text{in} \quad \Omega, \\ u &= 0 \quad \text{on} \quad \partial\Omega. \end{cases}$$

on a bounded and smooth domain $\Omega \subset \mathbb{R}^N$. In our approach F is a fully nonlinear elliptic operator (with certain structural conditions), q is a non-negative sub-linear absorption term in relation to homogeneity of operator, and $a : \Omega \to \mathbb{R}$ is a sign-changing weight. We also analyse some results concerning existence of non-negative solutions having dead cores.

Unique continuation principles for systems

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Abstract. In this talk I will discuss on some recent unique continuation results for systems of elliptic partial differential equations satisfying a suitable superlinearity condition. As an application, we obtain nonexistence of nontrivial (not necessarily positive) radial solutions, in the critical and supercritical regimes, for the Lane-Emden posed in a ball. Some of our results also apply to general fully nonlinear operators, such as Pucci's extremal operators, being new even for scalar equations. This is a joint work with Gabrielle Nornberg and Nicola Soave.

A limiting free boundary problem for a degenerate operator in Orlicz-Sobolev spaces

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Abstract. A free boundary optimization problem involving the Φ-Laplacian in Orlicz-Sobolev spaces is considered for the case where Φ does not satisfy the natural conditions introduced by Lieberman. A minimizer u_Φ having non-degeneracy at the free boundary is proved to exist and some important consequences are established, namely, the Lipschitz regularity of u_Φ along the free boundary, the locally uniform positive density of positivity set of u_Φ and that the free boundary is porous with porosity $\delta > 0$ and has finite $(N-\delta)$-Hausdorff measure. The method is based on a truncated minimization problem in terms of the Taylor polynomial of Φ of order $2k$. The proof demands to revisit the Lieberman's proof of a Harnack inequality and verify that the associated constant with this inequality is independent of k provided that k is sufficiently large.

On an abstract bifurcation result concerning homogeneous potential operators with applications to PDEs

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Abstract. We study an abstract equation in a reflexive Banach space, depending on a real parameter λ. The equation is composed by homogeneous potential operators. By analyzing the Nehari sets, we prove a bifurcation type result. In some particular cases we describe the full bifurcation diagram, and in general, we estimate the parameter λ_b for which the problem does not have non-zero solution where $\lambda > \lambda_b$. We give many applications to partial differential equations: Kirchhoff type equations, Schrödinger equations coupled with the electromagnetic field, Chern-Simons-Schrödinger systems and a nonlinear eigenvalue problem.

Ground states for a class of critical quasilinear coupled superlinear elliptic systems

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Abstract. We consider the following class of quasilinear coupled systems

$$\left\{ \begin{array}{ll} -\Delta u + a(x)u - \Delta(u^2)u = g(u) + \theta\alpha\lambda(x)|u|^{\alpha-2}u|v|^{\beta}, & x \in \mathbb{R}^N, \\ -\Delta v + b(x)v - \Delta(v^2)v = h(v) + \theta\beta\lambda(x)|v|^{\beta-2}v|u|^{\alpha}, & x \in \mathbb{R}^N, \end{array} \right\}.$$

where $N \geq 3$ and $a, b : \mathbb{R}^N \to \mathbb{R}$ are positive potentials, $\lambda : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative continuous function, $\theta > 0$ and $\alpha, \beta > 2$ satisfying $\alpha + \beta < 2 \cdot 2^*$. On the nonlinear terms we assume that g, h are in C^1 class which are superlinear functions at infinity and at the origin. We deal with nonlinearities g and h being subcritical or critical. The coupling term is a subcritical function which is superlinear at infinity. Our main theorem is stated without the well known Ambrosetti-Rabinowitz condition at infinity. Using a change of variable, we turn the quasilinear coupled system into a nonlinear coupled system, where we establish a variational approach based on Nehari method.

A limiting obstacle problem for the inhomogeneous $p-$fractional Laplacian

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Abstract. In this work we study an inhomogeneous obstacle type problem involving the fractional $p-$Laplacian operator (cf. Korvenpää *et al* [3]). Firstly, we establish existence and uniform estimates for any family of solutions $\{u_p\}_{p\geq 2}$ which depend on the data of the problem and universal parameters. Finally, we analyze the asymptotic behavior of such a family as $p \to \infty$. At this point, we prove that $\lim_{p\to\infty} u_p(x) = u_\infty(x)$ there exists (up to a subsequence), verifies a limiting obstacle type problem in the viscosity sense, and it is an $s-$Hölder continuous function (cf. da Silva & Rossi [1]).

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Fractional elliptic systems with noncoervice potentials

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Abstract. In this talk we study the existence of weak solution for a class of fractional elliptic systems of gradient type. The potentials are bounded from below and may change sign. One of them

can be also bounded from above. The nonlinearity term is smooth and it can be asymptotically linear or superlinear at infinity. We consider also an interation with the eigenvalues of the linearized problem and the nonlinear term. In the proofs we apply Variational Methods by considering both the resonant and non-resonant case. Our results are new even if we consider the local case.

An Indefinite Elliptic Problem on $\mathbb{R}^N$ Autonomous at Infinity: the Crossing Effect of the Spectrum and the Nonlinearity

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Abstract. We present a new approach to solve an indefinite Schrödinger Equation autonomous at infinity, by identifying the relation between the arrangement of the spectrum of the concerned operator and the behaviour of the nonlinearity at zero and at infinity. The main novelty is how to set a skillful linking structure that overcome the lack of compactness, depending on the growth of the nonlinear term and making use of information about the autonomous problem at infinity. Here no monotonicity assumptions is required on the nonlinearity, wich may be sign-changing as well as the potential. Furthermore, depending on the nonlinearity, the limit of the potential at infinity may be non-positive, so that zero may be an interior point in the essential spectrum of the Schrödinger operator.

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Posters

Normalized Solutions for a Schrödinger-Bopp-Podolsky System

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Abstract. Schrödinger-Poisson systems first appeared as the coupling of the Schrödinger's equation with a newtonian gravitational potential. Benci and Fortunato [1] studied a system of similar nature arising from the coupling of Schrödinger's equation with Maxwell's equations and, under Dirichlet boundary conditions, developed a now standard method to deal with this kind of problem. Later, Pisani and Siciliano [2] treated a Neumann problem for a similar system, where the procedure of Benci and Fortunato fails. In the present work we treat a modification of the problem studied in [2] which consists in the addition of a biharmonic term in the second equation (and imposition of appropriate boundary conditions):

$$-\Delta u + q\varphi u = \omega u$$
$$-\Delta \varphi + \Delta^2 \varphi = q u^2$$

in a smooth, bounded domain Ω subject to

$$u = 0, \qquad \frac{\partial \varphi}{\partial n} = h_1, \qquad \frac{\partial \Delta \varphi}{\partial n} = h_2$$

on $\partial\Omega$, where h_1, h_2 are continuous, $q \in C(\overline{\Omega})$, $\omega \in \mathbb{R}$ and $\int_\Omega u^2 \, dx = 1$. Motivation for the inclusion of the biharmonic operator comes from the Bopp-Podolsky electrodynamics, where the equation for the electric potential is of this form. We introduce a modification to the problem in order to deal with homogeneous boundary datum, outline the procedure by Benci and Fortunato and point out why it fails, present a technique to overcome this obstacle and prove the existence of infinitely many solutions $(u_n, \omega_n, \varphi_n) \in H_0^1(\Omega) \times \mathbb{R} \times H^2(\Omega)$.

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Existence of solutions for a nonlinear Schrödinger equation coupled with the Maxwell's equations

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Abstract. We seek to make a more detailed study of most of the ideas and results set forth in David Ruiz's [1]. For this, the following system of partial differential equations in $\mathbb{R}^3$ was considered

$$\begin{cases} -\Delta u + u + \lambda\phi u = u^{p-1} \\ -\Delta\phi = u^2, \quad \lim\limits_{|x|\to\infty} \phi(x) = 0 \end{cases} \tag{1}$$

where Δ is the Laplacian operator, $\lambda > 0$ is a real parameter and $p \in (2, 6)$. The system (1) is obtained by coupling a nonlinear Schrödinger equation with Maxwell's equations of electromagnetism (see [1] and references therein).

As the name suggests, the article deals with the Schrödinger-Poisson equation, which is a combination of two know equations: a Schrödinger equation - basis of quantum mechanics - and a Poisson equation, where a nonlinear place term u^p (or more often $f(u)$) has been added to this equation. These nonlinear terms are often used in the Schrödinger equation to model an interaction between particles. We give existence and nonexistence results, depending on the power p and parameters λ.

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The Lebesgue Measure: An Unmeasurable Set

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Abstract. One of the most important creations of Analysis of the last century was Lebesgue Integral, which remarkably extended Riemann Integral, solved within a few years the fundamental problems of Integration Theory which gave a relevant impetus to Functional Analysis, Theory of Differential Equations and Probability Theory. The basic point of this new theory was the introduction of the notion of measurement. The Lebesgue measure of $\mathbb{R}$ is roughly a function whose domain is a subset of $\mathbb{R}$ and whose contradiction is the set of nonnegative real numbers (joined with the symbol $+\infty$). The length of a range, for example, is a measure, say L, defined over all ranges of the real line, such that $L(I) = b - a$, where a and b, $a < b$, are the extremes of the range I and $L(I) = +\infty$ if I is not limited. Now, the L measure is defined for intervals only. It would be interesting to extend this concept to other subsets of the line. In this work, the outer measure of a subset of $\mathbb{R}$ will be defined and, with this measure, the notion of measurable set will be defined. Then the Lebesgue measure is presented. Our main goal is to build a set of real numbers that cannot be measured with the Lebesgue measure. We conclude that even extending the concept of interval length to other subsets does not encompass them all.
Keywords: Lebesgue, Measurable set, Real numbers, Measure.

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Existence of ground state solutions for superlinear and subcritical problems by the method of Nehari manifold

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Abstract. In this work, we present a study of the Method of Nehari Manifold treated in [1] which proposes a well-constructed theory to guarantee the existence of ground state solutions for some elliptic differential equations in Hilbert and Banach spaces. Here, we will apply the abstract results to the following equations:

$$\begin{cases} -\Delta u - \lambda u &= f(x,u), \quad \text{in } \Omega \\ u &= 0, \quad \text{in } \partial\Omega \end{cases} \quad \text{and} \quad \begin{cases} -\Delta_p u - \lambda |u|^{p-2}u &= f(x,u), \quad \text{in } \Omega \\ u &= 0, \quad \text{in } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ it's a bounded domain, Δ and Δ_p are the Laplacian e p-Laplacian operators, respectively.

Moreover, in the first equation we have $u \in H_0^1(\Omega)$, that is a Hilbert space, $\lambda < \lambda_1$ where λ_1 denotes the firts Dirichlet eigenvalue of $-\Delta$ in Ω, and $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$ satisfies the growth restriction $|f(x,u)| \leq a(1+|u|^{q-1})$ for some $a > 0$ and $2 < q < 2^*$, recall that $2^* := \dfrac{2N}{N-2}$ if $N \geq 3$ and $2^* := \infty$ otherwise.

In the second equation, we have $u \in W_0^{1,p}(\Omega)$, that is a Banach space, $\lambda < \lambda_1$ which λ_1 denotes the firts Dirichlet eigenvalue of $-\Delta_p$ in Ω, and $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$ satisfies the growth restriction $|f(x,u)| \leq a(1+|u|^{q-1})$ for some $a > 0$ e $p < q < p^*$, recall that $p^* := \dfrac{pN}{N-p}$ if $N \geq 3$ and $p^* := \infty$ otherwise.

Adding some assumptions on f we guarantee the existence of infinite pairs of solutions.

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Multiplicity of positive solutions for a gradient type, cooperative/competitive elliptic system

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Abstract. We study the existence of positive solutions for gradient type cooperative, competitive elliptic systems, which depends on real parameters λ, μ. Our analysis is purely variational and

depends on finer estimates with respect to the Nehari sets, in fact, we determine the extreme parameter $\lambda^*(\mu)$ for which the Nehari set is a manifold and hence standard variational techniques can be applied. We also discuss the case where the Nehari set is not a manifold.

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Dynamical Systems

Talks

The Topological Entropy of Powers on Lie Groups

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Abstract. This talk addresses the problem of the computation of the topological entropy of an application $\psi : G \to G$, where G is a Lie group, given by some power $\psi(g) = g^k$, with k a positive integer. When G is commutative, ψ is an endomorphism and its topological entropy is given by $h(\psi) = \dim(T(G)) \log(k)$, where $T(G)$ is the maximal torus of G, as shown in [1]. But when G is not commutative, ψ is no longer an endomorphism and these previous results cannot be used. Still, ψ has some interesting symmetries, for example, it commutes with the conjugations of G. In [2], the structure theory of Lie groups is used to show that $h(\psi) = \dim(T) \log(k)$, where T is a maximal torus of G, generalizing the commutative case formula.

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The Lie-Palais Theorem

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Abstract.

In this talk we present a more general version of the well-know Lie-Palais Theorem. The Lie-Palais Theorem, which is related to the fundamentals of Sophus Lie's theory, gives an answer to the following question: are the actions of Lie algebras consequences of the actions of Lie groups? This theorem was originally proved by Sophus Lie in a local context and later by Richard Palais, who proved the result as a global result. Specifically, Palais proved that the infinitesimal action θ of a finite dimension Lie algebra $\mathfrak{g}$ in a differentiable manifold M comes from a smooth action of a Lie group on M, if all vector fiels $\theta(X)$ are complete in M (see [2]).

As the years passed, this theorem was developed in lots of versions and applied in the most diverse situations. In our version, in the context presented in [3], we relax the hypothesis that all vector fields $\theta(X)$ are complete in M. There exists a proof (see [1]) for a result similar to the one we prove here, but they are different and produced in totally independent ways.

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The homogeneous Ricci flow on flag manifolds with three isotropy summands

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Abstract. For flag manifolds with 3 isotropy summands, we provides a complete description of the homogeneous Ricci flow: phase portrait and basins of attractions, including collapsing phenomena.

We use a dynamical approach that considerably simplifies the problem by appropriately projecting the Ricci flow in the positive orthant of $\mathbb{R}^3$ to a flow in the 2-dimensional simplex $x + y + z = 1$, so that Einstein metrics become fixed points of the projected flow. Then we use techniques from planar dynamical systems and Lie theory to proceed a case by case analysis of the phase portrait of the projected Ricci flow, for each flag manifold with three isotropy summands: there are 2 infinite families and 8 exceptional cases of such manifolds.

When the metric evolves to a degenerate one in the boundary of the above simplex, we show that the flag manifold collapses to another homogeneous space, in the sense of Gromov-Haussdorff convergence of metric spaces. Depending on the initial metric, the collapse happens either on the fiber or on the base of an appropriate projection from the flag manifold.

This is joint work with Lino Grama (Unicamp), Ricardo Miranda (Unicamp), Mauro Patrão (UnB) and Llohann Sperança (Unifesp).

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Dynamics of N-distal homeomorphisms of compact metric spaces

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Abstract. The distal homeomorphisms were introduced by Hilbert (p. 405 [2]) in order to generalize the isometries on metric spaces. Such homeomorphisms have been widely studied in the literature. For instance, Ellis [2] reduced them to the enveloping semi-groups and the minimal distal systems; Fustenberg [3] proved a structure theorem and Parry [4] proved that they have zero entropy (also derived from Fustenberg's). Generalizations of the distal systems include the point distal flows (by Veech [5] who obtained a structure theorem for them) and more recently N-distal homeomorphisms by

the authors in [1]. We will show some examples and dynamic properties of N-distal homeomorphisms, such as their connection with the equicontinuity, minimal and shadowing properties. Even more, we will prove that the topological entropy for minimal N-distal homeomorphisms is zero, which generalizes the results established by Parry in [4]. This is a joint work in progress with Elias Rego (UFRJ).

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Positive Ricci Curvature through Cheeger deformation

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Abstract. Let (M, g) be a Riemannian manifold with an isometric G-action. If a principal orbit has finite fundamental group and $\text{Ricci}_{M^{reg}/G} \geq 1$, Searle - Wilhelm proved that M admits a new metric $\tilde{g}$ of positive Ricci curvature. $\tilde{g}$ is obtained after a conformal change followed by a Cheeger deformation. The question remained on whether it is sufficient to consider only the Cheeger deformation to attain positive Ricci curvature on the new metric $\tilde{g}$. Here we approach this question by giving necessary and sufficient conditions on the G-action. In particular, we construct an infinite family of manifolds satisfying the hypothesis of Searle–Wilhelm that do not develop positive Ricci curvature after Cheeger deformation. Moreover, as a byproduct of the theory, we give simpler proofs to Searle - Wilhelm result and to Lawson - Yau theorem on the existence of positive scalar curvature on manifolds with non-abelian symmetry.

Minicourse

On the algebraic and smooth structures of the diffeomorphism group of compact manifold

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Abstract. The title seems more pretentious than it was meant to be. In this mini-course we will just introduce some notions of infinite dimensional Lie groups as well as their topology and representations. We will focus mainly on the natural action of the diffeomorphism group $\mathrm{Diff}(M)$ on a compact manifold M as well as on its right action by pullback on the bundle of symmetric covariant two tensors on M. Groups of diffeomorphisms of a manifold M have many of the properties of finite dimensional Lie groups, but also differ in many ways, for example, the exponential mappings are defined but are not locally surjective or injective. There are groups of diffeomorphisms which are smooth manifolds but only right translations are smooth. Surprisingly, the algebraic structure of the identity component $\mathrm{Diff}_0(M)$ determines the smooth structure of M. If $\mathrm{Diff}_0(M)$ is isomorphic to $\mathrm{Diff}_0(N)$, then M is diffeomorphic to N. Our interest in the present topic is motivated by the problem of finding a metric on a compact manifold with prescribed isometry group. We will show a theorem, due to Jean Pierre Bourguignon, that says that if G is a compact Lie subgroup of the group of diffeomorphism of the manifold satisfying a given algebraic property then there exists a metric g such that the isometry group is G. Motivated by this result we are currently investigating the related problem of finding a metric on homogeneous spaces with some more geometric or algebraic information prescribed. The prototype cases are compact Lie groups equipped with bi-invariant metric or more generally symmetric spaces.

This minicourse is organized as follows. We will start with a quick review of manifolds, Lie groups, G-action on manifolds, and we will finish this initial part with the Slice Theorem. In the second part, we will give the definition of a Hilbert Lie group as well as other weaker notions of infinite dimensional Lie group and some of their properties. The main goal of this part will be to show an analog of the Slice Theorem for $\mathrm{Diff}(M)$-action on the space of smooth Riemannian metrics of M. In the last part, we will show some applications to the problem of finding a Riemannian metric with prescribed isometry group.

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Poster

Hyperbolicity in One-dimensional

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Abstract. In this report we study consequences of hyperbolicity on discrete dynamical systems with domain on the circle S^1. We describe the chaotic behaviour on a specific hyperbolic set using symbolic dynamics and we study dynamical systems on $\mathbb{R}$ with hyperbolic periodic points the irrational flow on the torus.

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Geometry

Talks

Hamilton-Ivey Inequality for the Ricci-Bourguignon Flow

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Abstract. The goal of this talk is to present a result concerning solutions of a geometric flow, known as Ricci-Bourguignon Flow, which is useful in comparing the smallest and the biggest curvatures as time goes. Such inequality is known as Hamilton-Ivey Inequality and can be used to prove that certain special solutions have positive sectional curvature. The main obstacle relies on controlling a parameter ρ. The main result says that when this parameter is within $(-\frac{1}{2}, \frac{1}{4})$, the referred estimate is true and the solutions behave as solutions of the Ricci Flow.

Rigidity for self-expanding solutions of the mean curvature flows

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Abstract. In this talk, we discuss self-expanders which are the self-expanding solutions of the mean curvature flows. We give a universal lower bound of the bottom of the spectrum of the drifted Laplacian on a self-expander and prove an inequality between the bottom of the spectrum of the drifted Laplacian and the bottom of the spectrum of weighted stability operator. Also we prove some uniqueness properties of hyperplane for mean convex self-expanders.

Prescribing the curvature of Riemannian manifolds with boundary

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Abstract. Let M be a compact connected surface with boundary. We prove that the signal condition given by the Gauss-Bonnet theorem is necessary and sufficient for a given smooth function f on ∂M (resp. on M) to be geodesic curvature of the boundary (resp. the Gauss curvature) of some flat metric on M (resp. metric on M with geodesic boundary). For $n \geq 3$, we prove some topological restrictions which imply, among other things, that any function that is negative somewhere on ∂M (resp. on M) is a mean curvature of a scalar flat metric on M (resp. scalar curvature of a metric on M and minimal boundary with respect to this metric). As an application of our results, we obtain a classification theorem for manifolds with boundary.

Isometries and transformations of solutions for equations describing pseudo-spherical surfaces

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Abstract. Equations describing pseudospherical surfaces are characterized by the fact that their generic solutions provide metrics on a nonempty open subsets of $\mathbb{R}^2$, with Gaussian curvature $K =$

-1. These equations can also be seen as the compatibility condition of an associated $\mathfrak{sl}(2,\mathbb{R})$-valued linear problem, which is referred to as a zero curvature representation. In this talk we will discuss some new results on the use of isometries of pseudo-spherical surfaces to construct explicit solutions for such equations.

Geometry of submanifolds with respect to ambient vector fields

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Abstract. Given a Riemannian manifold N^n and $\mathcal{Z} \in \mathfrak{X}(N)$, an isometric immersion $f\colon M^m \to N^n$ is said to have the *constant ratio property with respect to* $\mathcal{Z}$ either if the tangent component $\mathcal{Z}_f^T$ of $\mathcal{Z}$ vanishes identically or if $\mathcal{Z}_f^T$ vanishes nowhere and the ratio $\|\mathcal{Z}_f^\perp\|/\|\mathcal{Z}_f^T\|$ between the lengths of the normal and tangent components of $\mathcal{Z}$ is constant along M^m. It has the *principal direction property with respect to* $\mathcal{Z}$ if $\mathcal{Z}_f^T$ is an eigenvector of all shape operators of f at all points of M^m. In this talk I will report on a joint work with F. Manfio and J. van der Veken, in which we have studied isometric immersions $f\colon M^m \to N^n$ of arbitrary codimension that have either the constant ratio or the principal direction property with respect to distinguished vector fields $\mathcal{Z}$ on space forms, product spaces $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$, where $\mathbb{S}^n$ and $\mathbb{H}^n$ are the n-dimensional sphere and hyperbolic space, respectively, and, more generally, on warped products $I \times_\rho \mathbb{Q}_\epsilon^n$ of an open interval $I \subset \mathbb{R}$ and a space form $\mathbb{Q}_\epsilon^n$. Starting from the observation that these properties are invariant under conformal changes of the ambient metric, we have provided new characterization and classification results of isometric immersions that satisfy either of those properties, or both of them simultaneously, for several relevant instances of $\mathcal{Z}$ as well as simpler descriptions and proofs of some known ones for particular cases of $\mathcal{Z}$ previously considered by many authors.

Isolated singularities of elliptic linear Weingarten surfaces

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Universidade Federal Fluminense

Abstract. In this talk we will focus on isolated singularities of elliptic linear Weingarten surfaces (i.e. a surfaces for which the mean and Gaussian curvature satisfy a relation as $aH+bK = 1$). We will do a local study by means of the classification of singular solutions of its associated Monge-Ampère equation. We will explain when and when not previous techniques concerning elliptic Monge-Ampère equations modeling convex surfaces apply to this more general case. This is a joint work with João Paulo dos Santos.

Volume growth for geodesic balls of static vacuum space on 3-manifolds

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Abstract. The purpose of this article is to study the geometry of static space–times. We show that the energy density and pressure vanish on the boundary of a perfect fluid space–time, provided that it satisfies a suitable condition. Moreover, we provide an upper bound volume growth for geodesic balls of the base of static vacuum space similar to a classical result due to Bishop. In addition, we derive a weak version of the maximum principle of Omori-Yau at infinity for such spaces. **Joint**

work with: H.Pina & E. Ribeiro Jr.

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Hypersurfaces with Constant Gauss-Kronecker Curvature in $M \times \mathbb{R}$

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Universidade Federal do Rio Grande do Norte

Abstract. In this talk, we will present some results from a recent joint work with João Paulo dos Santos and Fernando Manfio. We will consider hypersurfaces of general product spaces $M^n \times \mathbb{R}$ with constant Gauss-Kronecker curvature, which we call *K-hypersurfaces*. In this setting, we will address the following questions: Under what conditions (on M) does $M \times \mathbb{R}$ admit rotational K-hypersurfaces? Is there a general method for the construction of K-hypersurfaces in $M \times \mathbb{R}$? Are K-hypersurfaces of $M \times \mathbb{R}$ rigid?

Extrinsic geometric flows in Riemannian products

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Universidade Federal do Ceará

Abstract. In this talk, we present some existence results for extrinsic geometric flows of entire graphs in Riemannian products. If time permits, we will also discuss about mean curvature flows.

Surfaces in equilibrium under a external vertical force field

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Abstract. We construct examples of $[\varphi, \vec{e}_3]$-minimal surfaces of grim reaper type and classify them as the only complete flat $[\varphi, \vec{e}_3]$-minimal surfaces. We also prove the existence of bowl-type and catenoid-type examples and give in some particular cases its asymptotic behavior. Finally, we prove an uniqueness result for complete embedded $[\varphi, \vec{e}_3]$-minimal surfaces with a single end that are asymptotic to the bowl-type example.

Eigenvalue estimates for a class of elliptic differential operators in divergence form

Juliana Ferreira Ribeiro de Miranda (julianafrmiranda@gmail.com)
Universidade Federal do Amazonas

Abstract. In this work we compute estimates for eigenvalues of a class of linear second-order elliptic differential operators in divergence form (with Dirichlet boundary condition) on a bounded domain

in a complete Riemannian manifold. Our estimates are based upon the Weyl's asymptotic formula. As an application, we find a lower bound for the mean of the first k eigenvalues of the drifting Laplacian. In particular, we have extended for this operator a partial solution given by Cheng and Yang for the generalized conjecture of Polya. We also derive a second-Yang type inequality due to Chen and Cheng, and other two inequalities which generalize results by Cheng and Yang obtained for a domain in the unit sphere and for a domain in the projective space.

Rotationally Symmetric Solutions for Prescribed Schouten Tensor

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Abstract. We consider the Euclidean space $(\mathbb{R}^n, g)$, with $n \geq 3$ and $g_{ij} = \delta_{ij}$, and non-diagonal tensors $T = \sum_{i,j} f_{ij}(x) dx_i \otimes dx_j$. Assuming that the solutions are rotationally symmetric, we find the necessary and sufficient conditions for the existence of a metric $\bar{g}$ conformal to g, such that the Schouten tensor $\bar{g}$, is equal to T. From the obtained results, we show that for certain functions h, defined in $\mathbb{R}^n \setminus \{0\}$, there exist complete metrics $\bar{g}$, conformal to the Euclidean metric g, whose curvature $\sigma_2(\bar{g}) = h$.

Complete noncompact quasi-Einstein manifolds.

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UFC

Abstract. One of the main motivations to study quasi-Einstein metrics on a Riemannian manifold is its direct relation to the existence of Einstein warped products. Another important motivation comes from the study of diffusion operators by Bakry and Émery. In this talk, we will show that complete noncompact ($\lambda = 0$) quasi-Einstein manifolds are connected at infinity. Moreover, we will present volume growth estimates for complete noncompact quasi-Einstein manifolds similar to the classical results by Bishop, Calabi and Yau.

On the σ_2-curvature and applications

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Abstract. Given a Riemannian manifold (M^n, g), the σ_2-curvature is defined by

$$\sigma_2 = -\frac{1}{2}|Ric_g|^2 + \frac{n}{8(n-1)}R_g^2.$$

In this talk, we define a symmetric 2-tensor canonically associated to the σ_2 curvature, then we introduce the notion of σ_2-singular space and under certain hypotheses we prove a rigidity result[3]. After, we give a necessary and sufficient condition for a CPE metric to be Eisntein in terms of σ_2-singular spaces. Such a result improves our understanding about CPE metrics and Besse's conjecture with a new geometric point of view. Finally, we present an almost-Schur lemma for symmetric $(2,0)$-tensors on a compact Riemannian manifold of nonnegative Ricci curvature with totally geodesic boundary.

[2]The author was partially supported by PNPD/CAPES/Brazil.
[3]This work is joint with Almir Silva Santos - UFS

Posters

Superfícies Com Curvatura Gaussiana Constante em Espaços Conformemente Planos

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Abstract. Neste trabalho caracterizamos as superfícies de curvatura Gaussiana constante em espaços conformemente planos $(\mathbb{R}^3, \hat{g})$ onde $\hat{g} = \frac{\delta_{ij}}{F^2}$ e $F :\to \mathbb{R}$ é uma função radial $F(r) = F(x_1^2 + x_2^2 + x_3^2)$, consideramos $F(r) = \sqrt{(}r)$ e $F(r) = e^{-r^2}$ e construímos exemplos de superfícies completas com curvatura gaussiana zero em $(\mathbb{R}^3, \hat{g})$

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Application of a Bochner type formula for a large class of spaces

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Abstract. We establish a unified Bochner type formula for a large class of spaces, satisfying $f\mathring{Ric} = \mathring{\nabla}^2 f$, that include the static perfect fluid space-time, critical metrics of the volume functional, static spaces and CPE metrics. Moreover, as a consequence of such a formula, we obtain a result guarantees that the geodesic ball on a sphere S^3 is the unique compact static perfect fluid space-time with positive constant scalar curvature such that the norm of the Einstein tensor $|\mathring{Ric}|$ lies in the interval $[0, \frac{\sqrt{6}}{12}(\mu - 3\rho)]$, where μ and ρ are, respectively, the energy density and the pressure. Joint work R. Diógenes, B. Leandro, and E. Ribeiro JR.

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Rigidity of complete minimal submanifolds in a Hyperbolic space

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Abstract. We prove some gap theorem for complete immersed minimal submanifold of dimension no less than six or four, depending on the codimension, in a hyperbolic space $\mathbb{H}^{n+m}(-1)$. That is, we show that a high dimensional complete immersed minimal submanifold M in $\mathbb{H}^{n+m}(-1)$ is totally geodesic if the L^d norm of $|A|$, for some d, on geodesic balls centered at some point $p \in M$ has less than quadratic growth and if either $\sup_{x\in M} |A|^2$ is not too large or the L^n norm of $|A|$ on M is finite, where A is the second fundamental form of M.

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Generalized Weingarten Surfaces of the Radial Support Type

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Abstract. I intend to do a poster submission about surfaces that can be parametrized as envelopes of a sphere congruence whose other envelope is contained in a unit sphere. Such surfaces has a local parametrization given by

$$X(u) = Y(u) - 2\left(\frac{h(u)+c}{S(u)}\right)\eta(u) \quad u \in U, \tag{2}$$

where $h : U \subset \mathbb{R}^n \to \mathbb{R}$ is a real differentiable function associated with the parametrization $Y : U \subset \mathbb{R}^n \to \mathbb{S}^n$ of $\mathbb{S}^n$ and

$$\eta = \nabla_L h + hY, \qquad S = \langle \eta, \eta \rangle = \left|\nabla_L h\right|^2 + h^2,$$

with $L_{ij} = \langle Y_{,i}, Y_{,j} \rangle$.

In my work it is exhibited a suficient condition to exist such sphere congruence. Moreover its radius function is given explicitly and it is proved to be a geometric invariant of the surface.

The characterization of surfaces that are associated to $\mathbb{S}^n$ by a sphere congruence is used to study a class of generalized Weingarten surfaces, named generalized Weingarten surfaces of the radial support type - RSGW- surfaces in short - which satisfy a differentiable relation between the mean and Gaussian curvatures, the support function and the radius function from the sphere congruence.

Under certain condition, a surface locally parametrized as in (2) is a RSGW-surface if, and only if, the function h is harmonic. In this case, the vector function η is a local parametrization for a Appell surface. Futhermore, there is a Weierstrass type representation depending on two holomorphic functions for the RSGW-surfaces and, consequently, for the Appell surfaces locally parametrized by η.

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Mathematics Education

Talks

Mathematical and didactic knowledge of basic education teachers concerning patterns and regularities: reflections on continuing education based on practice

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Abstract. This communication seeks to investigate mathematical and didactic knowledge mobilized by mathematics teacher when preparing, developing and collectivelly analyzing a class on patterns and regularities in a high school class. For this, we present the context in which the research were developed, a process of training mathematics teachers that aimed mobilize and (re)build teacher's mathematical and didactic knowledge about patterns and regularities. The training process took place through professional learning assignments prepared in a way as to provide collective discussions on the subject in question. The research is qualitative-interpretative and the data was collected by recording audio and video, and through written documents produced by the participants. The results show that the professional learning assignments elaborated around the practice records of a class mistered for high school, as well as the collective discussions experienced by the participants, enabled the development of new professional knowledge in teachers regarding, for example, the collective work of planning classes and conducting collective discussions with their students.

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The use of routine and non-routine combinatorial questions and analysis of student records as reflection tools for teaching practice

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Abstract. The interest among Brazilian researchers in Mathematics Education increased in understanding the mathematical conception in High School, having as a tool to support the official evaluation system, such as the Basic Education Assessment System (SAEB). Thus, we defend the use of the results of an evaluation and analysis of the student records as pillars in the search for the reorientation of the teaching work. The present study aims to make use of the analysis of student records as a basis to identify the main strategies and difficulties presented by them, when dealing with combinatorial problems in routine and non-routine questions, as well as, to use the analysis of the records as an instrument of reflection for the teaching practice. The questions were selected in textbooks, used by the students at school and in previous editions of the National High School Exam (ENEM), in order to cover concepts from the fundamental counting principle to groupings. In an initial study, 246 students from 3rd year of High School of a public school in the Federal District. The use of calculators was allowed and the maintenance of attempts at the answers was encouraged. The results showed that the students were more successful in the routine question; they had low interpretative capacity in the non-routine question; they presented an interpretative and resolution capacity which was not compatible with the end of High School; resolution strategies were presented to elucidate the ongoing conceptualization processes and their meditational needs. A subsequent study was carried out with 91 students from the 3rd year of High School, in the same teaching unit, whose objective was to evaluate the students' change of posture, when they dealt with non-routine questions, outside a test situation and in a collaborative environment. The results revealed that when students are placed in an environment of interaction and dialogue, they present creative responses to non-routine questions and that, depending on how this type of question is presented, the initial results are changed.

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Assessment for learning in the Cálculo 1 course: the perception of students at a Brazilian public university

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Abstract. This research aims to analyze the perception of students about the Calculo 1 course, offered at a Brazilian public University. The research highlights the main aspects of the learning processs assessment and was based on the qualitative approach, with a phenomenological focus to analyze the organization of the didactic pedagogical work, the evaluation practice and some factors related with the success/failure of students on this course. A questionnaire was applied to 108 students of the part-time Calculo 1 course, 157 of the full time course of Calculo 2 in 2018 second semester and their answers were analyzed and helped to respond our questions in this study. From the data collects on these questionnaires, it was possible to analyze the closed questions using descriptive statistical elements such as: percentage and graphics. The opened questions were analyzed using the content analysis developed by Bardin, finding 16 categories out of the students. Lastly, from the data collected on questionnaires and the analyze of them, we can affirm that the failures of students on the Calculo 1 course is related to the assessment practices that doesn't favor an effective feedback to the students and by an organization of the didactic pedagogical work that doesn't favor the inner needs of those different courses.

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Assessment intervention in mathematics to the critical and creative thinking perspective: boundaries and possibilities

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Abstract. By promoting assessment strategies which takes into account the encouragement of the development of critical and creative thinking, the teacher who evaluates can move other bias not affected by the current evaluation system and have different results. Gontijo (2007) believes the execution of programs, training and use of creative techniques in the school environment can contribute to the development of students' creative potential. For Bailin (1987), critical thinking and creative thinking have distinct, but complementary characteristics: creativity is the ability to generate "intellectual" products; critical thinking is the ability to thinking carefully, strategically and critically about the quality of these "products". Taking into account such factors, in the present study, which is set up as an ongoing doctoral research project, we propose to investigate what are the boundaries and potentialities of the assessment interventions performed by the teachers in Mathematics classes in the early years, in the perspective of development of critical thinking and creative thinking, so that methodological strategies can be proposed that include formative assessment actions. For that purpose, we envision discussing about spaces, moments, approaches and possibilities offered to this aim, as ways of perceiving the formative assessment in the development of mathematical learning. In this research, of exploratory, bibliographic, observational and documental nature, we intent to gather of data through documental and bibliographic analysis and interviews with teachers. We also appreciate taking into consideration researches by some of the main methodological trends focused on the mathematics teaching-learning process, such as: teaching practices, conceptions and teacher's knowledge, assessment practices and the sociocultural context of mathematical learning (KILPATRICK, 1994 apud FIORENTINI and LORENZATO, 2006).

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Creativity workshop in mathematics: investigating the bodies' moves

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Abstract. According to Alencar and Fleith (2003), every human being shows creative potential, which needs improvement and training, through practice, appropriate techniques and favorable environmental conditions. Then, the Creativity grows from the not only individual conception, but obtained in the middle of a sociocultural process, considering the motivational factors that can lead the individual to dedication and involvement in what can give him pleasure and satisfaction (CSKISZENTMIHALYI, 1996). Based on these conceptions, we present the "Creativity Workshop in Mathematics", a project idealized and promoted by Grupo PI – Researches and Investigations in Mathematics Education, from the Department of Mathematics of the University of Brasilia. The project carried out an intervention with eight creativity workshops in the field of mathematics, in two classes of early years of a public school in the Federal District, in which the students were motivated to employ their own strategies to solve mathematical problems. In addition, another intentionality of the workshops was to place students in an environment favorable to the development of mathematics creativity, where playful and relaxed spaces replace the environmental pressures of the classroom (GONTIJO; FONSECA; ZANETTI, 2018). In this study we intend to analyze one of the workshops of the project in question: the workshop "Investigating the Bodies' Moves", in which the students designed and built a balloon-powered car to understand the scientific ideas related to rockets propulsion. They used also the mass and force ideas to discover ways to improve the distance traveled by the car. Therefore, we have tried to provide participating students with a playful, creative and interactive experience in Mathematics, aimed at building cars with sustainable materials.

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Lesson Study: Contributions to Teacher Education

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Abstract. The text deals with the contribuons and challenges of using the Lesson Study in the Brazilian teachers seng. It reflects on the main stages of this methodology, and how it has been

adequate in the development of teacher training projects, according to the specificies of each project. It discusses the possibilies of using this methodology at the micro level, but argues about possibilies of use at the macro level.

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Reflections on the teaching of mathematics in the context of interdisciplinary projects

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Abstract. For 20 years I worked as a mathematics teacher and sought to innovate through the use of technologies, investigative classes and various manipulative materials. In 2005 I also started to work on teacher training. At the Federal University of Itajubá, since 2013, I have been coordinating projects for the Teaching Initiation Scholarship Program (PIBID) and Pedagogical Residence (PRP), in addition to coordinating an Interdisciplinary Studies and Teacher Training Group (GEIFOP). In 2017 I also got involved with an Interdisciplinary Teaching Practice for the four university degrees, in which we approach the concept of interdisciplinarity (LAVAQUI; BATISTA, 2007) and some approaches. The programs, the discipline and the group bring together trainers from different areas, teachers of basic education and undergraduate students interested in discussing and developing innovative projects, of an interdisciplinary nature, to be developed at primary and secondary level. Realizing the wealth of projects developed in these areas, I have sought to investigate (CRISTOVÃO et. Al, 2018, CRISTOVÃO, 2019) the potential and limits of mathematics teaching in this context. In this presentation I will highlight two projects, one involving Mathematics and Dance (SANTOS, 2019) and the other Photography and Mathematics (in progress). Both were developed under the PRP, with support from Geifop.

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Mathematical Games, Gamification, and Game-Based Learning in Mathematics Education

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Abstract. An examination of uses of games in mathematics education, of combinatorial games in mathematics, and of gamification techniques in university courses. Aspects of the design and evaluation of games for the use in mathematics classrooms.

Mathematical education and citizenship: reflexes for a critical work direction

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Abstract. The present work, entitled "Mathematical Education and Citizenship: reflexes for a critical work direction"; as general objective, to raise the presence and the importance of the mathematics teaching in the formation of a citizen. Therefore, it can be highlighted that this work arose from a problem that aims to question what would be the function of this discipline besides using calculations and formulas, that old question raised by the students "Whom will you learn this"? This study seeks answers to the following questions: 1) What is the function of mathematics teaching besides using calculations; formulas; equations...? 2) How can it contribute to the formation of a critical citizen? 3) What is the role of mathematics teaching as an individual's discipline for society? It is noteworthy that research is exploratory, in which a literature review is used to collect information and field research to obtain data and empirical information, through a sample of ten teachers from the licensing courses in "Pedagogy and Mathematics". A university located in the vicinity of the Federal District, with the wrong statistics unfolding, both qualitative and quantitative. With this, you can understand how the relationship between Mathematics Teaching and Citizenship, and what is the view of the Teacher-Trainers in relation to it. In this respect, it is concluded that Mathematics

Teaching has an important role in the formation of the citizen and that it can and should contribute to the formation of a critical citizen. It is worth noting that they also noticed an alignment of the institution's teachers with these ideas.

Keywords: Mathematics Teaching. Mathematical education. Critical Citizen. Formation. Citizenship.

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Report of hybrid teaching experience applied in geometry teaching to middle school students

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Abstract. One of the teacher's great challenges today is to make his classes as attractive as the rampant bombardment of information, stemming from the technological advance in which society is immersed. Contrary to what one might think, it is not viable to combat the use of technological tools or resources by students, quite the opposite, combining these tools with the teacher's repertoire makes the teaching and learning process contextualized and contemporary. In this context, hybrid teaching emerges which consists of the combination of so-called traditional classes with information and communication technologies. According to Bacich and Moran (2015), hybrid means blended and the education is mixed or hybrid, combining various spaces and activities for learning. This work aims to report a hybrid teaching experience with students of middle school, who participate in an extension project promoted by the Department of Mathematics of the Federal University of Amazonas. As claimed by Bacich and Moran (2015), in the hybrid teaching there is an organization of four main models: rotation, flex, à La Carte and enriched virtual. The object of knowledge addressed was Geometric Solids and the chosen model was the rotation one, which is divided in workstations so that each station had an objective well defined by the teacher, linked to the objective of the class. Each group passed all the stations within a time defined by the teacher. According to the theory, at least one station must be involved with the use of technology (ANDRADE; SOUZA, 2016). The activity was developed in four classes, in a total of six hours/class and were developed by academics from

the Mathematics Licenciate Degree course which participated from the creation to the application of classes and acted as advisors in the processes of building students' knowledge. From the observations made during the application of hybrid teaching, we could perceive the involvement of students in the proposed activities, demonstrating the potential these methodologies have to transform the learning process into an active experience, where students are active and, consequently, collaborative subjects in their learning development.

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Mathematics and Art: dialogues under construction

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Abstract. This lecture aims to present the initial research on the possibilities of intersection between mathematics and art as a way to combine the rational and logical learning of mathematics with the sensitive aspects of art. The research is being carried out based on a pertinent bibliographic reference based on the proposal of Herbert Read (2013) that points to the Platonic perspective that Education and Art must go hand in hand in order to allow the full development of sensitivity. In addition, the research proposal uses the historical approach of Mathematics and Art presented by Zaleski Filho (2013), going deeper into the work of the artist Piet Mondrian (1872-1944). As a specific objective in the construction of this dialogue between Mathematics and Art, there is a confluence with Ethnomathematics, through the work of Paulus Gerdes (2010) addressing the construction of drawings called "sonas" by the Cokwe people of Angola.

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Math teaching for ADHD students

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Abstract. Attention Deficit Hyperactivity Disorder (ADHD) is a developmental disorder that shows characteristics of inattention, hyperactivity and impulsivity, which can accompany the individual throughout his life. Thus, it arises the desire to study and analyze the difficulties faced by students who have this disorder in this course completion paper. Therefore, the guiding question aims to question teachers and students from 6th to 9th grade of elementary school from three public schools in Formosa-GO, seeking to investigate their difficulties about teaching and learning mathematics and if they use appropriate methodologies and resources. The goal of this work seeks to perceive how the teaching-learning process occurs at targeted students with ADHD. And the specifics aim to: a) understand the difficulties of teachers and students with ADHD in the mathematics teaching-learning process; b) identify the methodology used by the teachers towards these students; c) recognize the concrete teaching resources that assist in learning and concentration of students with this disorder. For this reason, this paper was unfolded in an exploratory way, with the unfolding of a descriptive research, making a bibliographical review. Afterwards, field research was carried out, where questionnaires were used as an instrument of data collect. Throughout the research, we realized the need to use methodologies that are more interesting from the student with ADHD point of view, applying teaching strategies and didactic resources that make the student concentrate and seek to learn Mathematics, making the process of teaching-learning easier, as this will help teachers have no difficulty in teaching mathematics to these students who will also have an easier time learning the subject.
Keywords: Attention Deficit Hyperactivity Disorder. Teaching. Inclusion. Difficulty. Mathematics.

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Inclusive Mathematics Education and professional development: building school-university partnerships

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Abstract. Inclusive mathematics education acknowledges human diversity and involves supporting the diverse learning needs of all students in general mathematics classrooms. But, in the teacher education rarely opportunities exist for the understanding and development of the inclusive mathematical practices. I describe here some projects that intend create a school-university partnership in which teachers, future teachers and researchers together build alternatives and teaching proposals aimed at a mathematical education that includes everyone.

The Lesson Study (LS) in mathematics licentiate degree: initial studies at the University of Brasilia (UnB)

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Abstract. The Mathematics Licentiate Degree, in Brazil, coexist, on the one hand, with the lack of perspectives for the teaching career due to the pau- perization, prevarication and proletarianization of the teaching work and, on the other hand, with the classic dichotomy between disciplinary knowledge and pedagogical knowledge, in the training processes. Facing historical ad- versities in the formation of the Mathematics Licentiate Degree of UnB, and in search for knowledge and dialogues in favor of its (re)construction and, consequently, of its practice of training mathematics teachers. The learnings coming from the Institutional Program of Teaching Initiation Scholarships (PIBID/Mat/UnB) through the construction of Mathematics Learning No- tebooks and the studies in Hybrid LS (HLS) on professional development of teachers and/or future teachers collective training constructs motived us in the development of the current study. Thus, the objective is to understand the training processes and the learning for the teaching of mathematics licen- tiate undergraduate when coursing the discipline of Supervised Internship in Middle School (MS), having as methodology the glocal LS. To this end, twelve undergraduate licentiate, two training professors of UnB, two mathe- matics teachers of Secretary of State for Education of the Federal District (SEEDF), one Federal Public School teacher and two private school tea- chers, composed, by consensus, four subgroups for studies and completion of a HLS cycle. The curricular topics worked embraced a signi cant part of Middle School from sixth to ninth grade, ranging from equivalent fractions to trigonometry in the right triangle. All performed the following actions: choice of curricular topic, studies, initial planning, collective planning, te- aching among the undergraduate licentiate, class analysis, (re)planning for teaching with the students from MS, teaching with the students of MS, class analysis, critical discussion of the classes and produce narrative analysis. The actions were documented through written register, audio recording and video recording of the work meetings at both school and university. For the recording, we had the technical support of professionals of the area, what allowed the team to dedicate themselves fully to the pedagogical actions. As partial results, we observed: 1/ the widening of the dialogue among all the participants in the planning, analysis and (re)elaboration of classes; 2/ the production of oral reports by all involved about theoretical and metho- dological aspects observed during the classes; 3/ the expansion of studies in mathematics teaching, especially, on the National Curriculum Common Core (BNCC) and the conceptual domains in focus; 4/ di culties in de - ning the role of the observer during classes; 5/ preference for oral reports to written narratives, among

other aspects. In addition, in both subgroups, the identi cation by some undergraduate licentiate of di erent mathematics reasoning among the students stands out, and the di erentiation between mathematical and didactic pedagogical objective in the context of develo- ped tasks. As di culties, were identi ed: the inexperience of the group in collective actions; the speci cities of constructing investigative tasks in the curricular topics in focus and the mediation of speci c conceptual demands of MS students.

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Teachers Education to Teach Mathematics from a Decolonial Perspective

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Abstract. In recent decades, several Latin American authors in the field of social and human sciences have referred to coloniality as a pattern of power that emerged and survives from modern colonialism, operating through the naturalization of racial and social dichotomies and hierarchies, on three interconnected axes: power, knowledge and being. Decoloniality then emerges as an epistemological and political position that shapes postures and projects of resistance, transgression, intervention and insurgency against coloniality. In this talk, I propose some reflections on how this decolonial perspective can contribute to push the debate on teachers' education, in particular teachers who teach mathematics, to a political terrain, seeking to reveal traces and effects of colonial relationships that are mobilized within contexts of teachers' education and practices. In particular, I highlight three intertwined dimensions for this discussion: relational, ideational, material and cultural references; intersubjective relationships and micro-politics; methodologies and epistemologies.

Critical and creative thinking in mathematics: a concept under construction in the Brazilian context

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Abstract. The development of critical and creative thinking is increasingly emphasized on the international stage as a necessary skill for the 21st century. In Brazil, this subject is still something new. Based on the literature review, the Brazilian curriculum standards and the results of some empirical research, we will present a theoretical framework related to critical and creative thinking and ways to stimulate this type of thinking in the field of mathematics.

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Sources of Information in Research on the History of Mathematical Education: A Pentadimensional Approach

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Abstract. In the present work, reflections related to the five dimensions about Mathematics Education Research: Ontological (What is called Historical Sources?). Epistemological (What is the relationship that the historian of Mathematics Education establishes with its sources as evidences that support their claims, working hypotheses, conjectures, conclusions, etc.? Teleological (What is the purpose of having sources of reliable information in the studies of History of Mathematical Education?); Axiological (What is the value that should be attributed to the sources of information in the studies of History of Mathematical Education?) And Methodological (What strategies are suitable to form Corpus -based on reliable sources of information- in the research on the History of Mathematics Education? What are the most appropriate techniques for analyzing content of research sources in research on History of Mathematics Education?)

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Conception of a mathematics teaching laboratory: from implementation to first activities

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Abstract. The present text addresses the results of a research that aimed to use teaching methodologies using didactics material from the Mathematics Teaching Laboratory (LEMA) that help in the expansion of the Mathematical knowledge of Basic Education contents linked to students' day by day problem situation. To this end, we investigated the process of conception and implementing a LEMA, and the teaching activities development to a first year class of High School of a western Bahia school. The research has a qualitative character, since this type of investigation allows subsidizing the procedures and information seeking to understand the student's behavior, studying their particularities and individual experiences, focusing on the problem under study. The methodology adopted was the action research, where the researcher is a research participant and allows qualifying his/her practice through an intervention. As research development, we elaborated and developed an Action Plan divided into have activities where were worked methodologies involving problem solving, didactics materials, history of mathematics and computer as the use of software as didactic resource for teaching the mathematical content of the Cartesian plan. As theoretical reference we are based on Fiorentini; Lorenzato (2006), Lorenzato (2009), Luckesi (2014), Mendes (2009), Smole; Diniz; Milani (2007), Rêgo; Rêgo (2009). Preliminary results allow us to infer that the use of different didactics-methodological resources in teaching promoted attribution of meaning to the content worked for the students and a reorientation in the practice of the teacher-researcher. The teaching became slower initially, however in the sequence of didactics content, there was a compensation in time and mainly in quality, because the pace of comprehension was faster due to the acquired comprehension.

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Chess game in the mathematics teaching and learning process: creativity, problem solving and deductive logical reasoning

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Abstract. This paper exposes the execution of an extension action carried out at UEG-Goiás State University, Campus Formosa. The action provided theoretical and practical studies involving the Chess Game in workshops, championships and olympics, with a view to stimulating deductive logical reasoning, creativity, patience and interaction between those involved (UEG students, high school students and community members). Thus, it is noticeable that schools are the main drivers in the development of skills that can be explored in relationships that arise with the game practice, which occurred in categories and patterns, in view of the manipulation of the chess developed practice, which contributes also for the individual formation. Based on authors such as Becker (2002), Capablanca (2008), Caldeira (2009), Gontijo (2007), Gontijo et. al (2019), Lubart (2007) and others, this action sought a qualified study involving the Mathematics undergraduate students and High School students from a Formosa scholl. The methodology had been organized in three fundamental stages: Theoretical studies (through reflections in workshops and lectures); Literacy of those involved in the game (UEG students, colleges and local community); and, Conducting workshops, theory classes, championships and Olympics of the Chess Game (at UEG and high school). Therefore, it can be stated that the symbolic game exercises the imagination, just as when playing the Chess Game, and it is believed that the use of this game as a teaching strategy, in a dynamic, competitive and pleasant way, promotes, improvements in the teaching and learning process, especially regarding creativity and problem solving in mathematics.
Keywords: Teaching. Learning. Creativity. Logical reasoning. Mathematics.

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Ethnomathematics in the School Curriculum

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Abstract. From the socio-historical-cultural, theoretical-philosophical and political-educational perspectives of the Program Ethnomathematics an analysis of Basic Education Curricula of Public Schools in the Federal District - High School stage, from 2000 to 2018, is performed. Using the dimensions of Program Ethnomathematics as investigative object and the concepts of *Curriculum Trivium* to inform characteristics and contributions of the epistemological-cognitive conception of this Program to the Curricular Proposals of Public Schools in the Federal District.

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Mathematics: teaching, knowledge and recreations through history

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Abstract. Our professional and bibliographical experience has shown us a number of evidences of

deep links in the mathematical context between recreation, teaching and knowledge, including the notions of rigor and validity of a theory. Moreover, when one of these three elements is in focus, the other two often emerge at some point. Thus, we believe that despite the strong resistances still presented today in the academic context of mathematics, especially by teachers at all levels, the quintessential place of recreation is in the context of mathematics, either in the production of pure mathematical knowledge or in the teaching process. and learning. Therefore, we aim in this text to make some reflections on Mathematical Recreations, Mathematical Knowledge and Mathematical Education throughout history, trying to establish some convergences and relationships between these three categories.

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CINEMAT: mathematics through cinema lenses

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Abstract. This communication aims to foster a reflection on the cinema's potential as an instrument of mediation for a mathematical enculturation of the 21st century youth. We present our findings regarding the research project *CINEMAT: mathematics through cinema lenses* which has been taking place at Federal University of Viçosa involving undergraduate students. The project is based on the Theory of Activity (1978) which sustains that learning happens through actions and interactions within a given culture. Real world activity is the means by which man develops his psychic abilities. The development of individuals' abilities and skills happens through the appropriation of the historical-cultural legacy of the preceding generations, through a social and communicative process, mediated by relations with others. Mathematics occupies a prominent place in human history and culture and, this way, mathematical learning becomes a necessity for the full exercise of citizenship in our days (Moreira, 2016). On the other hand, Bishop (1991) argues that mathematics is a cultural phenomenon that transcends the boundaries of society analogously to music or sport. In this sense, cinema allows us to use image for a historical, sociocultural and personal reconstruction of mathematical objects. Audiovisual language favors the approach of mathematical knowledge while culture portrays its social function. It enables the comprehension of societies through expressions and it creates a track record for the posterity of their social practices, modes of thinking, values, symbols, feelings, behaviors, tendencies, expectations and fears (Carvalho, 1998). Cinema acts as an mediating instrument for a mathematical enculturation of students by stimulating positive attitudes on mathematics, making learning more dynamic, interesting and attractive. Some selected films were exibed. Each session was followed by discussion dynamics about the mathematical and cultural strategies covered in the film. As sources of data, questionnaires were answered by participants. Results indicate that cinema favors the appropriation of mathematical objects and a perception of mathematics as part of human culture.

Keywords: cinema; mathematical enculturation; theory of activity.

Integration of research experiences and continuing education of early years teachers

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Abstract. This work present partial results of an integration experience between three training proposal that focused on the continuing education of Early Years of Elementary School mathematics teachers of the municipal school system. The authors, training professors and researchers in the area of Mathematics Education, met periodically to analyze the statements of early years mathematics teachers that participated in different training proposal: PNAIC 2017/2018, managed by the State Secretariat of Education of Territorial Center of Education 11 (NTE 11), research-training project developed during the years of 2016 and 2017, as eld of study of the Doctorate Course in Education and, in 2019, the continuing education program of the Municipal Secretariat of Education. This analysis aimed to identify the contributions of training proposals in the production of knowledge, by teachers who supported the adoption of differentiated work practices in the classroom, specifically in mathematics classes. The joint work of the researchers-trainers made it possible to follow changes in the organization of the teachers' pedagogical work, as they, in the dynamics of the training process, became authors of this process, indicating their training needs and ways more coherent with their realities for the implementation of the proposals in the classroom. It is intended that this experience be transformed into a research-training as advocated by Longarezi and Silva (2013) and, for this purpose, it is given more priority to promote training more than to collect data, adopting the principle of exibility regarding the content investigated at each moment and, teachers are proposed situations in which they can develop and produce their own knowledge and feel challenged to study educational theories that contribute to the understanding and explanation of their practices. In this context, the teachers collaborated with their own training, suggesting, questioning, registering the students' actions, evaluating their cognitive development, analyzing the results of their work. The next step is to intensify the registering of actions in the classroom by both the students and the teachers in order to produce narratives that are moments to think about the work and to direct discussions about the construction of their professional identity.

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Cooperative games and the inclusion process in Mathematics teaching

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Abstract. The genesis of games is connected to the processes of development of man itself. So much so that Huizinga (2017) affirmed that it is in the Game and through the game that civilization emerges and develops. Games are found in culture, in war, in law and in jurisprudence. In all these applications the game becomes a mediator of interactions and of knowledge construction. According to Brugére (1997), an universal definition for games cannot be found in the literature. Thus, researchers who assess and analyze these tools should build and defend a concept of Game themselves. In our own understanding, we should not define games, but rather characterize them. In this sense, games can comprise both a free play activity (voluntary), carried out within certain limits of time and space; and activities with rules that should be accepted and enforced by the players; they are endowed with an aim in themselves and accompanied by feelings of tension and joy (HUIZINGA, 2017). Games can be categorized as competitive or cooperative. In competitive games you play against the other while in cooperation you play with the other. In Mathematics teaching the game is gradually introduced and its use has gained importance among educators and within an emancipatory pedagogical praxis that presents itself as a need for students to effectively take action their process of knowledge building. In this context, the objective of this communication is to discuss the contributions of cooperative games for the inclusion of students with Specific Educational Needs in the teaching of Mathematics. We will present the results of a systematic literature review for the research developed in this field in the past five years. We highlight some of the characteristics described in cooperative games, which may favor the process of educational inclusion, such as: the achievement of individual goals which are, partly, a consequence of the action of other members and the fact that players become more sensitive to each other's requests, promoting mutual help and balancing contributions and participation.

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Mathematical learning of students with intellectual disabilities: Research Field

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Abstract. The purpose of this paper is to map the academic production about mathematical learning of students with intellectual disabilities in Brazil, as well as the analysis of methodological design, objectives and results Gil (2010). The theme was defined as mathematical learning of the student with intellectual disability and the period between 2000 and 2019. The research sources were the repositories of the universities, the Brazilian Institute of Information on Science and Technology (IBICT) and the Capes Portal Thesis and Dissertation Bank. The main search criterion was to find a relationship between a term on Mathematical Education and one on Intellectual Disability, necessarily mentioning both. This procedure aimed to constitute the research field of mathematical learning of people with Intellectual Disabilities. Methodological aspects were analyzed and recurrences, convergences and divergences in the methodologies could be verified. The qualitative approach was predominant, and it was possible to realize the need for rigor in the description of the method, as well as the epistemological base positioning. Regarding the results, the research pointed learning possibilities and advances in didactic-pedagogical aspects, aiming at the development of mathematical learning in the inclusive perspective.
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Teacher training and thematic evaluation: duo sometimes distant

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Abstract. As part of a doctoral research, a questionnaire was applied to teachers of mathematics in basic and public education of Brasilia. This excerpt present data on professional experience, initial and continuing education of these teachers and adhesion to the Formative Evaluation proposal, which is the officially recommended perspective. Just over half of the respondents say they take up this proposal in their classrooms. Despite the absolute majority having a degree in the area in which they teach and having invested in their continuing education in the last five years and/or in graduate school, the topic of evaluation of learning is not recurrent, 49% of them only saw the topic in one of their training and for 11.4% of the respondents the topic never constituted an object of studies.

Teachers adopt a variety of evaluative tools and procedures, but to those who adhere to formative evaluation there is greater number of tools and procedures used. The collected point to possible fragility regarding the training of those who teach mathematics in the approach to evaluate their students' learning in the basic education classroom.

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The mathematics teaching process in elementary school early grades at Formosa – GO public schools

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Abstract. The developed study approaches the process of mathematics teaching in the initial and final grades of elementary school public schools in Formosa-GO, which offer elementary school in the final grades, whose lags were observed on mathematical prerequisites. From this demand, the present research starts from the following problem question: is the learning difficulty during the elementary school initial grades related only to the practices developed by the teachers or also to their mathematical formation? The overall objective is based on the analysis of the relationship between the difficulties presented by 6th grade students and the numbering process. For this, it was necessary to know the curriculum matrix of the Mathematics discipline of the Pedagogy course, to analyze the Mathematics Teaching Plan regarding the practices and content applied in the Elementary School of the initial grades, to verify the contribution of Mathematics teaching in the grades to the students' difficulties in the development process in the following stages of education. The methodology adopted

is the descriptive with qualitative focus. The research instruments were questionnaires directed to the elementary and junior high school teachers. The results of the questionnaires show that most teachers in the initial grades do not feel mathematically prepared to work the subject, for this reason, many times they do not deep the content for lack of security, but they use diversified practices in their classes. As for the teachers of the 6th grade, they report a lag in numbering, which hinders the continuous development of the mathematical knowledge construction for this stage of teaching. Although the teachers in initial grades accomplish continuing education, according to the research, the mathematical approach during the pedagogy course, pursuant to the matrix, does not contemplate enough to them feel safe in the execution of mathematics classes. The research had as theoretical background mainly on BNCC (2018), D'Ambrósio (1989 and 2013), PCN (1997), PCN: Mathematics (2001), Romanowski (2012) and Sebach (2018).
Keywords: Mathematics; Formation; Teacher; Student.

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Curricular progression and the teaching of algebra in middle school: an analysis of the abilities proposed in the National Curriculum Common Core (BNCC)

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Abstract. This documental research analyzed the abilities presented in the knowledge area of Mathematics, in the thematic unit of Algebra, proposed by the National Curriculum Common Core-BNCC (2017), focusing on investigating the curricular progression during Elementary and Middle School. The National Curriculum Common Core is a the document that regulates which are the essential learning to be worked in Brazilian public and private schools of Elementary, Middle and High School. The analysis was made from three categories: cognitive taxonomy, for which the reviewed Bloom's Taxonomy was chosen; the object of mathematical knowledge, studied from theoretical references such as Canavarro (2007) and Gusmão (2009); and the proposed space of literacy. This work was excecuted from the necessity presented by the Basic Education teachers, in continuing education, of understanding the perspective of teaching introjected in the BNCC document, from the inclusion of this new thematic unit in Early Years of Elementary School and its alterations proposed in final years of Middle School.

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Model generation by logiscal regression to predict and explain the approval or failure in discipline

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Abstract. Researchers debate the best practices to solve the problem within the educational scope that, despite the availability of a large database that management has, it is necessary to translate them into information that is useful for implementation of public policies [3]. The present study addresses this issue in the following aspect: how to use modern analytical techniques to identify students with a high risk of failing, aiming at greater effectiveness in the directing resources to the most vulnerable group. The issue of evasions reflects not only academic performance, but also a range of personal, economical aspects and institutional factores [2]. Therefore, highlighting the different mechanisms that contribute to the student's success can guide educational managers on the implementation of public policies [1]. In addition, the construction of tools capable of identify precociously the greater risk of evasion would refine the school and educational intervention programs. Thus, the following questions become congruent: Which elements and/or conditions explain or assist in the understanding school success and how can we measure it by statistics tools? Is it possible to develop a model that is sensible to the point of perceiving these conditions aiming preventive action from the teaching institution? Considering this problem, this work consists of understanding the factors that lead to approval/failure in the Fundamentals of Mathematics (FM) component of the Mathematics Licenciate Degree Course of the Federal Institute of Brasilia (IFB)–campus Estrututal. To this end, it was executed the organization of data, Exploratory Analysis, Inference and construction of the best explanatory and predictive model, based on Cross Industry Standard Process for Data Mining (CRISP-DM) methodology. With the available data, we randomly divided 70% as the model's training base and the other 30% as test. We applied Logistic Regression using the R language reaching the following results: the variables that were significant together to explain the approval/failure effect were region (naturalness) and number of failures of students in FM. The constructed model performed well, reaching an accuracy of 94.17% in the training stage and 88.64%in the testing stage and is consistent with others modeling tests given the evaluation metric. The area of the ROC curve was of 96.3% in training and 90.4% in test. The results and their interpretation will be presented. It is considered an initial study that will serve as pedagogical support tool in the future for the Mathematics Licenciate Degree Course, Federal Institute of Brasilia (IFB)–campus Estrutural, promoting preventive actions against failure, one of the indirect factors of school evasion. A posterior

study with a larger number of data and refinement of the model will enhance the interventions and implementation of public policies of educational management. This work was carried out under the guidance of professor Pedro Carvalho Brom (IFB).

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Mathematics Teachers Interpretative Knowledge and its specificities for teaching – a need for a change of focus in teacher education

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Abstract. Mathematics teachers' Education (teachers initial and continuous training) needs to focus on the particularities of teachers practices when compared with others that use mathematics as a resource (e.g., engineers). In that sense there is the need for specific focus of attention when discussing the mathematical topics in order to contribute for the development of teachers knowledge related with the work of teaching they are expected to perform. Such mathematical knowledge – which is considered Specialized – grounds the knowledge that will allow them to efectivelly have the students own reasoning, knowledge and habilities as a starting point for the mathematical discussions to occur in the classroom. Such knowlwdge is termed Interpretative Knowledge. Grounded in some examples from school mathematics I will discuss the specialized nature of such Interpretative Knowledge and some dimensions of its relationship with the more advanced mathematical knowledge tipically discussed at a more "advanced" level but usually without connections with the future teachers mathematical practices. Such discussion aims at problematize also our own knowledge as teacher educators (including all of us who teach some courses in the degree to become mathematics teachers) and the need for a change of focus in teacher education and in our own practices as teacher Educators.

Developing mathematical knowledge for teaching algebra: an innovative approach for teacher education courses

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Abstract. Recent literature indicates the need for research about the genesis of professional learning of mathematics teachers for teaching mathematics topics at school. In order to investigate this problem, we have been organizing teacher education courses focused on algebra, aiming to develop and expand teachers' mathematical and didactical knowledge regarding school algebra. These courses have been carried out during ten to fifteen 4 hours weekly meetings, composed of moments of (i)

individual work, (ii) work in small groups, and (iii) plenary collective discussions. The participants, pre-service and in-service mathematics teachers, developed hands-on activities, which were mediated by tasks elaborated by teacher educators. The main goal of the research is to understand how the professional learning of the mathematics teacher to teaching school algebra is developed. These courses are innovative, first, because it creates opportunities for teachers to learn from each other, thus breaking the traditional isolation of their work and expanding their opportunities to learn collectively; second, because it allows teachers to experience an exploratory learning environment built from classroom practice and mediated by (i) professional learning tasks, (ii) collective mathematical discussions, and (iii) the role and actions of the teacher educator. The theoretical framework is supported by the concept of didactical knowledge, and from a methodological point of view, this research follows a qualitative-interpretive perspective, through design-based research (DBR). Data are collected using participant observation with audio and video recording and document collection. The results show teachers (i) reorganizing their knowledge of algebraic concepts (e.g. functions) and expanding their capacity of representing and articulating different forms of representing these concepts; (ii) deepening their reflection about difficulties that the students find, for example, with the concept of a function and about teaching resources and strategies to overcome those difficulties; and (iii) being aware of their own difficulties regarding the concept of a variable, what could implicate the recognition of patterns and regularities and the formulation of algebraic expressions to represent complex mathematical sequences. We expect that this research will contribute to mathematics education by (i) favoring the presence and interlocution of practice as an essential component in teachers' mathematical and didactical knowledge to teaching algebra and (ii) rethinking approaches for teacher education courses.

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Differencial and integral calculus for students with high skilss/giftedness

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Abstract. In this paper we present the results obtained during the extension project entitled Differential and Integral Calculus (CDI) for students with High Skills / Giftedness (AH / SD), from GISNO resource room, which currently have 36 elementary and high school students. The students were encouraged to study CDI and its applications, which are part of the curriculum of certain undergraduate courses at the University of Brasília (UnB) in order promote academic enrichment and stimulating the creative potential of AH / SD of them. Weekly meetings were held with expository theory classes and classes on solving exercises and application exercises. In the application exercises, we addressed some problems that can be solved using the current curriculum of High School and also the curriculum of CDI, providing a link between High School and Higher Education. In the interest of checking whether the proposed objectives were achieved, two questionnaires were carried out, the first, six months after the start of activities and the second at the end of the project. The first questionnaire consisted of a discursive evaluation of the course, that is, a qualification about the interaction of the teacher teachers with the students, as well as a self-evaluation of the performance and participation in the CDI classes previously held. The questions sought to promote the reflection of the students behavior in class and allow a more active participation in the learning process, represented here by a column where they could write suggestions for the second half of the project. The second questionnaire was divided into two parts. The first is a repetition of the previous questionnaire in order to verify whether the suggestions adopted were positively received and confirm the students impressions about the course. And the second, an evaluative questionnaire with four questions each referring to one of the contents covered. With that in mind we compiled the students answers and analyzed them in order to obtain the results here presented.

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On the teaching of pre-calculus and its impact on differential and integral calculus

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Abstract. It is noticeable that students enter the university with disabilities in learning basic mathematics content. This implies, among other things, important difficulties in learning the content presented in the discipline of Differential and Integral Calculus (CDI), which is usually offered in the first semester of the courses. This difficulty contributes to the increase in dropout and failure rates in CDI. With the aim to reduce these negative impacts, it was offered a course that covered Pre-Calculus content in the two freshman classes, night and day, of the Mathematics course at the University of Brasília (UnB), in the second semester of 2019. This course consisted of weekly meetings during one hour and fifty minutes. In each of them, it was revised the content of high school that would be necessary to understand the theory of CDI to be presented in the following week. Students had weekly assignments that were delivered and corrected by monitors. The tasks were short so that the time of resolution and study did not hinder the activities of the CDI course. The results were measured by comparing the average grades of the CDI exams of these two classes with that of the other classes of CDI at UnB. This was possible by the fact that the CDI course at UnB is unified, so that all 19 classes take the same test. The final approval rate was also purchased, with very good results. A third analysis made was the improvement of students' learning in the basic subjects of high school. For that, evaluative tests were applied at the beginning and at the end of the Pre-Calculus course. In general, all students had an improvement in learning.

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Perimeter and area within the scope of special educational needs – vision impairement

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Abstract. Inclusive education is legally guaranteed nationally and internationally. In Brazil, the right to inclusive education is based, for example, on the Child and Adolescent Statute (Law nº 8,069 from 1990) and in the National Education Guidelines and Framework Law (1996). Internationally, there is the Word Declaration of Education for All (1990) and the Salamanca Statement (1994), among others. These documents establish that students with special educational needs must receive specialized assistance, as well as that inclusive teaching should preferably take place in the regular school system. For this reason, it is necessary to prepare schools to receive these students and,

therefore, it is essential that the teachers are able to offer specific attention to the needs of these students. To this end, it is necessary that there are didactic support materials that make possible a greater degree of understanding of the contents to be taught, minimizing the limitations faced by these students, which happens in different degrees. Among these students with special educational needs we have the visually impaired (DV), the focus of this research. This work aims to present a tactile didactic material, developed by two volunteers and guided by two professors of the Federal Institute of Brasilia (IFB)–campus Estrutural, from a PIBIC (Institutional Scientific Initiation Scholarship Program) project linked to PIBID (Institutional Scholarship Program for Initiation in Teaching). Such material was thought and built, with the intention of teaching the perimeter and area contents to visually impaired (DV) students. It is also intended to report the experience of application of a didactic sequence, using the material built, made in a primary school in Brasilia to two DV students. The material was made from MDF wooden plates in which geometric cutouts (square and rectangle) were made, so that the DV students could fit units of golden material, each unit has one centimeter of edge, inside each geometrical shape. This way, it became much simpler, to DV students, to find the measurements of the figures worked, as well as to understand and absorb, in fact, the content taught. The sequence was applied with the purpose that the students to learn a mathematical content associating it directly with their dailyives, thus applying, a real sense to what was worked on. The sequence and the didactic material developed showed a result of enormous satisfaction, since it reached all the determined goals and, beyond that, it brought us greater perspectives for an evolution of the developed sequence.

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Reflections on Mathematics PIBID: its potentialities and its challenges in face of teachers formation

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Abstract. The basic formation of math teachers and their teaching practice has uneasy the community of mathematical educators and has been widely investigated, generating substantial debates and many publications. In D'Ambrósio (1993), Fiorentini and Cristovão (2006), Ponte, Quaresma and Branco (2012) we found examples of studies on this movement that contribute to the understanding the possibilities, difficulties and challenges related to this formation. In this sense, programs as PIBID are initiatives for the improvement and the valorization of training of basic education teachers. The subproject "PIBID Matemática Goiânia-UFG" is developing collective, theoretical and practical actions that improve formative experiences for the graduates and others involved people, based on the dialogue and permanent exchange between graduates, teachers and schools students. Our actions

promote the effective and intense contact of graduates with the school reality and its specificities, and the expansion and consolidation of mechanisms and competencies inherent to the performance and training of math teachers. On the one hand, the subproject is characterized as a single opportunity for the formation of new teachers (graduates), besides for the continuing education of school teachers and undergraduate teachers (tutors); on the other, promotes the development of spaces for active reflection and upgrading of teaching practice, culminating in the awakening of school students to enjoy math and understand that it interacts with their lives. In view of this, we propose some reflections, which refer to: i) the need to promote formative actions for the insertion of graduates in the school environment parallel to their theoretical-practical formation in the university; (ii) the possibilities and potentialities of investigative practices; iii) the maintenance of projects like PIBID and the survival of these in face of the limitations faced.

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Didactic engineering as research methodology and educational proposal in mathematics

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Abstract. The following research is cut out of a doctoral research which analyzed the results of a didactic sequence that involved a ludo-didactic collaborative activity (LISBOA, 2019) [1]. The activity focused on the learning of the point and line to students of the third year of public high school of the Federal District, in the capital of Brazil. This is a case study and a participatory research that used the Didactic Engineering (ARTIGUE, 1996) [2] as a methodology of research and as an educational proposal in mathematics. Brum and Schuhmacher (2013) [3] warning that the Didactics of Mathematics is not reduced to researching the best way or teaching method for a specific idea or scientific concept. In the same way, Artigue (1996) [2] mentions that a scientific investigation methodology is needed to gather the relations between research and action in the system based on pre-established didactic knowledge. Therefore, the Didactic Engineering as a methodology is characterized as a didactic object that involves a teaching plan, the creation of didactic materials and an experimental scheme, all based on the didactic situations of the classroom (ARTIGUE, 1996; BRUM, SCHUHMACHER, 2013) [2] [3]. Thus, we present the development of the Didactic Engineering along with the accomplishment of the constituent phases of the method.

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Contributions of Lesson Study and Didactical Engineering for the Education and Professional Development of Preservice Math Teachers

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Abstract. This project is a research proposal that is being developed in the doctorate and has as general objective to analyse the contributions between elements of Lesson Study and Didactical Engineering (MIYAKAWA; WINSLOW, 2009), adapted to the Brazilian institutional context, to foster the professional development of teacher education in relation to the field of Quantities and Measures (LIMA; BELLEMAIN, 2010). Didactical engineering (DE) (ARTIGUE, 1994) is a methodology for experimental research in mathematics education, with a strong root and base in the Theory of Didactic Situations (BROUSSEAU, 1997). Collaborative LS has been achieving positive results in several places in the world, and involves three to four phases: lesson planning, research-lesson, reflect phase, and in some cases the revision of the lesson plan (BAPTISTA et al., 2012). It can be used with teachers and/or pre-service teachers. Then, the field of Quantities and Measures was chosen for its potential to connect with social practices, other Mathematics contents as well as with other disciplines. Some results will also be presented.

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The chess game in the math learning process in a Formosa state college - Goiás

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Abstract. The purpose of this paper is to verify whether there are benefits in using the chess game as a factor of considerable importance in the teaching-learning process of mathematics. An analysis that requires observation and reflection on this teaching and learning method, also recognizing the importance of the Chess Game in the school and social environment. Nonetheless, this study aims to prove the best from chess game in this method in a playful and applied way, allowing the students from the school involved, a broader view of what is Mathematics and its features, promote simulations of problem situations with diverse methods for learning math. In this sense, it is aimed to understand the stimuli which the application of the Chess Game performs concerning the teaching-learning process of Mathematics in students from a state school in Formosa - Goiás. And with this objective, the three aspects to be observed in specific objectives were triggered and it was expected to: a) Recognize ways to acquire knowledge, which are different from the traditional format, using the Chess Game; b) Identify how to bring down the barriers regarding mathematics being difficult or impossible and that it is not possible to learn by using chess game as a teaching methodology; c) Encourage the practice with the theory in the classroom by using the Chess Game. To achieve that, authors such as Becker (2006); Moreira (2019); Vygotsky (2007) among others supported this study with theory. It is worth mentioning this study has a bibliographic nature, applying a mixed, qualitative and quantitative methodology, by using the following instruments: questionnaires and semi-structured interviews and student observation. Thus, it was applied the chess Game to a sample of students, aiming to analyse the development of the teaching-learning process of theirs. Therefore, it was possible to conclude clearly that the use of chess in the teaching-learning process in Mathematics is crucial since it brings a significant meaning to the cognitive development of the students, and it is with this view and with the purpose to obtain results that the reflections and questions in the course of this study made the chess game be seen as a tool that supports in the teaching-learning process in math.
Keywords: Learning. Teaching. Chess game. Mathematics.

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Creativity in the perspective of 6^{th} students of a public school

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Abstract. This paper discusses the creativity in perspective of elementary school students, and aims to analyze the concepts and definitions of creativity from the point of view of 6th grade students, where it relates to the concepts already existing in the vision of scholars in the area. The importance of the theme for both school and social development is highlighted here. This study starts from a bibliographic and field review, with a qualitative and quantitative research. It is also

worth mentioning that you use the quiz application to search for relevant data. In the field research, questionnaires are applied to 25 (twenty-five) students of the 6th grade of Elementary School II, from a public school, in order to recognize what is their point of view regarding creativity and what are the your next perspectives on classroom creativity. Therefore, it can be concluded that this work has relevance on the reflexes of the creativity processes and the difference from the point of view of the students in relation to the scholars of the area, where there may be important reflexes. In this case, the results will be shown at the end of the analysis, being made as comparisons of collected data, it was possible to perceive the students' view of the evidence, evidence of how general creativity is the creation and innovation of something, and is present in all that exists. Each time you look for more creativity people in our daily lives, this shows the importance of creativity in our life. **Keywords**: Creativity. Creativity Concepts. Learning.

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Lesson Study as a Paradidactic Infrastructure for Development of Mathematics Teacher Knowledge

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Abstract. Lesson Study is a format for teachers work outside the classroom which originated in Japan, in which a group of teachers, often guided by a university researcher, designs and experiments a "research lesson" in view of gaining knowledge about the teaching of a specific mathematical subject, as well as on more general aspects of teaching. It has since been transposed to other countries. In my talk, after a general introduction to the nature and characteristics of this format, I will report on results from research on cases from Japan, Denmark and Brazil, drawing on joint works with T. Miyakawa, J. Bahn, P. Baltar and A. Dias, respectively

Minicourses

Technology to Learn, Study, Teach, Apply Mathematics

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Abstract.
In this lecture I present some possibilities of using technology in Mathematics Education. I approach the solving of real problems presented in papers and textbooks in which I use GeoGebra and/or Scratch. In resolutions are mobilized arithmetical, algebraic and geometrical knowledge, exploring ways of mathematical and computational thinking based on what is proposed in the problems' statements. In addition, I propose a way of using GeoGebra/Scratch to unfold the problems addressed into more complex ones.

Towards a Teachers Pre-Service Education Built from within the Profession

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Abstract. In this workshop, I present and discuss some curricular components from an institutional policy currently conducted at the Federal University of Rio de Janeiro (UFRJ), designed for pre-service mathematics teachers' education at undergraduate level, from a conception of teachers' education as a professional education. This conception is grounded on a political stance of teaching at school level as a professional activity, with its specific knowledge and practices, which must be integrated with but cannot be subordinated to academic mathematics knowledge and practices. Therefore, undergraduate teachers' education is regarded as a professional education, in which knowledge emerging from practice is formally incorporated, and above which school teachers' author it is formally acknowledged.

Exploring Mathematics through dialogic and investigatory learning

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Abstract. Mathematics can be an exciting and mind-opening topic in the classroom. The concepts of Dialogic and Investigatory Learning enable teachers to create mathematical activities in which the students could get in touch with specific mathematical thinking and problem solving. In the workshop we will present some theoretical framework related to the theme and give practical examples for different education levels.

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The SAIAM method of mathematical applications: multiplication, percentage, potentiation and division

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Abstract. The SAIAM MATHEMATIC APPLICATIONS method is a method of building tables that was created and developed by a self-taught scholar and researcher author of this work. This method was designed to enable an individual to learn, more briefly, from the 3rd and 4th years of elementary school, the operations of multiplication, division, potentiation. Still based on the construction of tables, the method also works as an auxiliary tool to solve problems with these operations, percentage, positive integers and decimals. The table-building method is considered to differ for the better from the traditional one by having only 32 characters, two operators and an algorithm while the traditional one has 270 characters, which can make your learning traumatic. In this work, we will show how the method works, its advantages and the results of the research done so far about it. Motivating results will also be presented, such as the possibility of reducing the time required for learning the cited content and the positive responses of those who participated in previous research, ie children and teachers. We will also present some examples of building tables and also some resolutions of the operations and applications mentioned.

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Responding to some "mathematical whys" through Dynamic Mathematical Demonstration

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Abstract. Mathematical proof is essential to the mathematical understanding process. In general, in Basic Education, although textbooks contain mathematical proofs and suggestions for formula

deductions, they are still little used by teachers. Perhaps because for them the evidence of understanding is more related to the ability to solve exercises or problems, using mathematical properties. In that case, students may find it sufficient to memorize the formula or property in order to solve the problems. In many cases, failure to understand the deduction or demonstration process may lead the student to not know how it applies to properties or theorems. In this sense, the objective of the mini-course is to explore Dynamic Mathematical Demonstrations (NÓBRIGA, 2019) to explain some "mathematical whys".

Keywords: GeoGebra, Dynamic Mathematical Demonstrations.

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School Geometry For Elementary Course: Knowing The Past, Reflecting On The Present

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The workshop aims to present and discuss tasks adapted from manuals for teaching geometry in the early years (which were the subject of research for the production of a history of school geometry) over the years. Different authors, with different backgrounds and from several countries, developed and produced textbooks in order to drift apart from the formal approach of Euclidean Geometry, searching to elaborate an intuitive and experimental geometry to lead teachers in their pedagogical practices. For this workshop, four manuals were selected, two from the 19th century and two from the early 20th century to analyze different approaches and proposals for teaching a particular topic: the square. Participants are invited to solve the proposed exercises (adaptations of the textbooks in question) and to analyze the pertinence or not of inserting such tasks in their classrooms. The teachers are supposed to make a reflection about the creation and reformulation of knowledge in schools over the years, due to the pedagogical movements that circulate internationally. Reflecting on the past allows that teachers have a more critical overview of the current reality.

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Active Methodologies in Mathematics Class: theoretical and methodological aspects

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Abstract. The rapid technical and technological development has raised questions on the role and importance of the school, as well as teacher in this "new context" of "ultra-connected society". Contrary to what it may seem, recent research shows that the relevance of the teaching professional has been increasing, precisely because the teacher has a multiplicity of specialized knowledge that are necessary for quality pedagogical results. The question then is not the importance of the school and the teacher, but to understand how it can be provided in the continuous teaching professional development to work in the context of permanent change. In this workshop, we will work with two teaching methodologies: specialized groups with rotation of stations, developed by the author, and project-based learning. In both cases, we will address the potentials and limitations of these methodologies, based on the reflections provided by their application with the group.

Mathematics for life

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Abstract. "But teacher, what am I going to use this for in my life?" You probably have heard, or asked, this question. Truth is that this often has no easy answer. In this workshop, we will work specifically with High School content that are considered by students to be hard and show why these themes are part of basic education, how they can be fundamental for scientific/citizen formation and how they are applied in academic and professional contexts. With this, we want to help the participants to understand the importance of mathematics in their lives, making them autonomous in the search for these answers and authors to share questions, hypothesis and their own answers to the initial questions.

Posters

Reflection of a graduate student in mathematics on the impacts of fomentation programs on teacher training

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Abstract. Educators and specialists from different areas defend the idea of training the "teacher-researcher", the term deals with the critical and reflective professional, autonomous and active, capable of reflecting on their practices and improve them. In this sense, research has the role of help in academic and professional training, in a liberating way, making the future professor capable of improve his/her teaching work more and more. The arguments in favor of the development of research are strong: a higher level of professional autonomy, increased ability to solve problems in the classroom; motivation for the exercise of teaching; development and improvement of curriculum; production of results communication, among others. "What a teacher has of a researcher is not a quality or a way of being or acting that adds to that of teaching. It is part of the practical nature of the teacher to ask, to seek, to research, what is need is that, in his permanent training, the teacher perceive and assumes himself, because a teacher, as researcher" (FREIRE, 1996, p. 32. Free translation). Stating these values, it is worth mentioning the importance of incentives and financing related to research during the academic training and professional performance of government and institutional programs. Without such incentives, research often becomes a reclusive practice, in which only those with greater capital are able to improve professionally. In this regard, I intend to cite incentive programs in which I participated and participle during my graduation, such as: Institutional Scholarship Program for Initiation in Teaching (PIBID), Institutional Scientific Initiation Scholarship Program (PIBIC), Scientific Technician, Technical Visit, among other. Reporting also, how much they added to my training. Since my second semester of graduation, I have been working in a line of research of inclusive education, with participation in two PIBIC projects I was able to develop methodologies and didactics material for the teaching of mathematics for visually impaired students, through PIBID I was able to have educational experiences in real environment, in order to apply the knowledge acquired and improve it. In calls for Scientific Technician in which I was awarded, I had the opportunity to disseminate the results acquired in others projects, as well as to socialize with other researchers, in the same way Technical Visits provided me work in other institutions, conquering new knowledge that I can disseminate in the research groups present at Federal Institute of Brasilia (IFB) – Campus Estrutural.

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Amazonian mathematics fair: giving life to mathematics in the middle of the forest

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Abstract. Over the years of experience with the teaching of Mathematics in Basic Education, it is evident, that for most students of public schools, notice Mathematics present in their daily lives is a great challenge. The teaching practice decontextualized is an important factor to be considered, as it leads to student's lack of interest in the discipline, making it meaningless for everyday life. To the detriment of this lack of interest, early on, the student, in the initial phase of his trajectory in Basic Education, builds a self-image of incapacity in relation to the discipline (PONTE, 1994). This problem present in metropolis schools, such as Manaus, acquires a much larger dimension in schools in the inland of the State of Amazonas and gains more expression in rural communities of these inlands. Giving the above, the objective is to divulge the extension action called Amazonian Mathematics Fair, carried out by the Mathematics Department of the Federal University of Amazonas, in partnership with the Federal Institute of Education, Science and Technology of Amazonas and the University of the State of Amazonas. The Mathematics Fairs have the purpose of giving a new meaning to school activities, transforming them into true laboratories for active experience of scientific learning, co-participated by the community and, thus, deconstructing the elitist vision of the teaching and learning process of Mathematics (ZERMIANI, 2014). It is a summary of the two edition of the Fair, held so, highlighting the challenge of carrying out a pedagogical and didactic action of Mathematics at state level, in the State of Amazonas, with all its social, political and economics peculiarities, not losing sight of its geographical challenges. It will be exposed, in this work, how the movement of Mathematics Fair in Amazonas has instigated new perspectives for teaching of Mathematics and provided new spaces for reflection and action. In the Amazonian Mathematics Fair, as well as in other Mathematics fairs held in the country, everyone is valued and the main driver is socialization and exchange of experiences (FRONZA et al., 2016). In addition, it is written here, how this project has been an instrument of inclusion, by involving students with special needs, inserted in the education of young people and adults, indigenous schools and rural communities, contributing to these students learning of Mathematics, differently from the current and instituted in many Mathematics classes. It was found that the participation of these students in this extension action, has been leading them to a process of increasing their self-esteem, in order to reframe their potentials, giving them the opportunity to express them according to their skills. There was a change and transformation in the lives of the students who, before the participation in the project, did not envision perspectives for overcoming their challenges in relation to the construction of their own knowledge; after the action they are even aiming to enter university. **Keywords:** Teaching practice. Knowledge and learning. Mathematics Fair.

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Active learning methodologies and their contribution to differential and integral calculus teaching

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Abstract. The Differential and Integral Calculus discipline has been, historically, responsible for high rates of failure and evasion, in many degree courses of exact and natural sciences. Among the justification for this fact, the deficient teaching in basic education, the lack of motivation on the part of the student and the unsatisfactory professor-student interaction, are in the studies on this problematic. Other than these, the inadequate teaching methodology is also considered responsible for this situation, since the traditional teaching conception, which permeates the exact sciences degree courses, does not meet the needs of the current social and educational context. In this conception, also called technicist, the accumulation of information for later reproduction it is valued, which is understood as insufficient to promote the student's success in this discipline. One of the possibility for teaching, in a different perspective from the technicist one, are the active methodologies that can be understand as ways of developing the learning process, used to conduct to the individual's critical formation, favoring their autonomy, wakening the curiosity and stimulating decision making. The present paper presents the doctoral thesis research project, which aims to validate active learning methodologies in Differential and Integral Calculus, in order to promote the student learning. The approach proposed for research is qualitative, with an approximation to the phenomenological method, since our intent is to research the reality through its manifestations, without restricting us to quantifiers. The investigation will take place with students of this discipline, in a public institution of higher education, with the execution of activities planned and conducted by didactic engineering, which is characterized by a scheme based on didactic achievements in the classroom, and on case study records. We believe that active methodologies can contribute to the development of the subject as a whole, provide autonomy, stimulate creativity and prepare them to face new and challenging situations.

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The curricula of mathematics licentiate degree of the University of Brasília (UnB), since its creation until the present day

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Abstract. This poster aims to present the different curricula of Mathematics Licentiate Degree of the University of Brasilia (UnB), since its creation in the decade of the 60s of the last century until the present day and it is part of the research entitled The History of the Constitution of the Department of Mathematics of the University of Brasilia – MAT/UnB. It was used as research source, official documents that are under custody of the Department of Mathematics of UnB, laws and curricular orientation on teachers training in Brazil. The following questions were raised: How was created the first curriculum of Mathematics Licentiate Degree of the University of Brasilia? Which were the official documents that supported the construction of this curriculum? Which conceptions of education and teachers' training were the basis in the moment of its construction? What transformations have this course suffered over the years? What were the motivation that caused these transformations?

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Numerical sets - didactic sequence for visually impaired students

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Abstract. From a bibliographic study and participation in Institutional Scientific Initiation Scholarship Program (PIBIC) and Institutional Scholarship Program for Initiation in Teaching (PIBID) projects, it was requested to elaborate a didactic sequence to visually impaired students, with the main objective the understanding and inclusion of these students in the classroom. At this point, when analyzing one of the papers presented, the idea was to create a pedagogical material inspired by the one presented, but with a different content. It was sought a content in which there were not many didactics materials already created, such as geometry, and that was important for the curriculum of the discipline of mathematics, therefore, it was chosen the content of numerical sets. Initially, in the development process of the sequence, two types of materials were created: Material 1/ common hula hoop and one with string around it that would be used in classes; Material 2/ a hose cut in several circles of different sizes and plates with numbers in Braille. In addition, five classes were elaborate which, according to the materials to be used, were divided in two groups: group A and group B. Group A has the sets as content and uses material 1 for classes 1 and 2. Group B, on the other hand, has the numerical sets as content and uses the material 2 for classes 3, 4 and 5. Group A: In view of the concepts to be present and using two hula hoops, it will be explained to the students the relations between the sets through the method of Venn-Euler diagram, in which the sets will be represented by the hula hoops and the students will be the elements belonging to it. Group B: Using the plates with numbers and the circles with different sizes, the difference between the sets of natural, integers, rationals, irrationals and reals numbers and the need for creating each of them will be presented, as well as the relation between them. After the application of the didactic sequence it was concluded that it is a very effective toll for teaching visually impaired students, as it also facilitates the understanding of the content, makes the students work in group, interacting with their classmates, thus obtaining inclusion of the student in the classroom.

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The mines of Mathematics

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Abstract. The objective of this project is to promote mathematical knowledge exchanges with an exclusive target on women audience, of all ages, to improve the capacity of learning, enabling progress in school performance, both in mathematics and in other subjects. Encourage the personal growth of each student, giving them tools to improve their self-esteem and professional and academic growth. The classes involve contents of History of Mathematics, Logical reasoning, Problem solving and approach to professions under the eyes of women. The meetings take place weekly on Wednesdays for an hour and a half. Each meeting is divided into three moments: the first, the theme deals with empowerment; in the second, activities involving Mathematics contents are worked on; and finally the theme is about professions. As a stimulus, to the permanence and enjoyment of the meetings, motivational prizes donated by local traders are offered. The evaluation of the results achieved by the project in the participants' school routine is done by consulting the school record.

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The use of active methodologies in the development of skills and competencies in the teaching of mathematics

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Abstract. This work presents the result of the development of a Didactic Sequence of the curricular component of Mathematics applied in classes of the 8th year of the final series. The work was developed in a private educational institution located in Brasília, in the Federal District - Brazil. Our objective here was to demonstrate, through the application of a didactic sequence based on the assumptions of the active methodology, through the use of investigative, experimental, dialogic and object exploration practices, that students become autonomous and protagonists of the teaching learning process, moments that favor the development of mathematical skills and competences.
Keywords: Following teaching. Active methodologies. Student protagonism.

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Efective impact of university extension: the Institute of Exact Sciences (IE) during the university week at University of Brasília (UnB)

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Abstract. It is known that one of the great commitments of the university is to constitute itself a space/time for production and dissemination of knowledge as well as social inclusion. Thus, university extension at the Exact Science Institute (IE) of UnB is understood as an interdisciplinary, educational, cultural, scientific and political process. It is argued that it can re-signify the interaction between the university, its students, its professors and the community in general. This way, the IE rebuilt its extension policy based on three pillars: 1/ integrate students and professors from its courses (Mathematics, Computer Science, Computer Engineering, Mechatronics Engineering and Statistics) in the production and promotion of extension actions; 2/ offer these actions regularly to students and teachers from Basic Education (EB) and the community in order to reconstruct negative social representations, still in force in our society, which points the exact sciences as inaccessible to all, especially to girls and women; 3/ evaluate its actions in order to build new knowledge and improve future offers. Therefore, in this paper, it is reported an evaluative process carried out during the XVIII University Week of UnB (SU), in 2018, aiming to understand the impact of university extension on the school and personal life of three groups of participants, namely: EB students and teachers who participated in the offered actions and monitors (IE undergraduate students who helped in offering and organizing IE's participation in the SU). To this end, the three groups were invited to collaborate voluntarily by answering questions, organized in electronic forms, such as: age, gender and education level; previous participation in extension actions; evaluation of the university's functions and responsibilities; evaluation of interest in the different areas of knowledge, in particular, those that compose the IE; evaluation of the activities offered and their impact on future professional choice, among others. Data analysis favored the quantitative and qualitative treatment of the responses provided, seeking to comprehend the impact factor in the participants' understanding. The results show that the EB teachers perceived the SU as a pedagogical tool of excellence to reframe the content already worked in the classroom, as an element of motivation to awaken scientific nature interests among students and teachers. As for the EB students, the data reveal that they perceive the SU as an opportunity to access information on: areas of knowledge, scientific research, career, labor market, professional remuneration and social ascension. Monitors, on the other hand, perceive the SU differently, according to their area. Licienciate undergraduates from the last semesters, for the most

part, assess that the SU presents them with the future professional field by creating opportunities for mediating concepts with the EB students; among the bachelors undergraduates, they perceive the SU as field of professional development due to the access to research, to public policies and to the identification and resolution of problems demanded by the community. The study is configured as an important indicator that will greatly assist the internal of evaluation and management policy in extension of both IE and UnB.

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Mechanics

Talks

Turbulent clustering of low-inertia droplets: an important process in the dynamics of clouds

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Abstract. Several effects of turbulence may reduce the time for rain initiation in warm (ice-free) atmospheric clouds. This talk overviews the effect of turbulent clustering of inertial droplets that interact hydrodynamically. Using the so-called drift-diffusion model (based on the Master equation satisfied by the probability of finding two droplets separated by a given distance) it is shown that hydrodynamic interactions have strong impact on clustering at small (sub-Kolmogorov) length scales [1]. This enhances droplet collision probabilities and may accelerate precipitation formation. Mechanisms for droplet clustering at larger length scales (in the inertial range of isotropic turbulence) are also discussed [2].

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Numerical modelling of fluid-structure interaction phenomena: from direct simulation to stability and sensitivity analyses.

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Abstract. Fluid-structure interaction is present in many natural and engineering systems. This type of interaction has great importance in a number of energy conversion and propulsion devices, and at the same time it is responsible for vibration and motion of structures that can ultimately lead to failure. Therefore, understanding the physical mechanisms that drive the interaction and predicting the responses of the structure and the flow may enable more efficient design and control strategies. Numerical modelling and simulation has been largely employed as a tool with this objective. However, the coupling between the fluid and structure often poses new challenges whose solution require novel methods and approaches. In this talk, a general framework to understand and analyse fluid-structure interaction problems will be given, and the main numerical methods used to model and simulate them are going to be presented. In addition, state-of-the-art tools to assess stability, sensitivity and receptivity of fluid-structure systems will be introduced.

Applied mathematics in Petroleum Engineering

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Abstract. Some analytic studies on filtration combustion will be presented. We will consider mathematical models describing the phenomenon in different physical configurations. Some examples

that will be considered: the existence and uniqueness of solutions was considered for combustion in foams and in porous media, taking into account the thermal losses. The models are composed of Partial Differential Equations (Balance Laws). The corresponding Riemann Problem solutions are presented as a sequence of contact waves and traveling waves using techniques from Conservation Law Theory and Dynamical Systems. Analytic estimates are validated through numerical simulations using Finite Element Method.

Steady Flow of a Uniform Rivulet Down a Vertical Wall

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Abstract. We investigate the steady flow of a uniform rivulet down a vertical wall. This problem has many applications, which are easily found in everyday life, such as paint running down on a canvas and chocolate syrup flowing over a cake. In nature, examples can be seen in the lava flow from an erupting volcano and melting glaciers. Initial studies, such those of Towell & Rothfeld [1] and Allen & Biggin [2], employed a thin-film approximation to determine the free surface profile, as a function of the static contact angle, and the velocity field. Duffy & Moffatt [3] have also studied the problem within the framework of the lubrication theory. In the present study, we have mapped the physical domain (free-surface profile) onto a rectangular computational domain and then solved the full problem (Navier-Stokes equations) numerically using a finite difference method. The code was validated with an exact solution presented by Perazzo & Gratton [4]. Furthermore, a new lubrication approximation was formulated using gradient dynamics based on the recent work of Lopes et al. [5]. The results were compared with the ones obtained previously in the literature and it was found that the new developed model is not only the most accurate among the reduced models, but also that it works well beyond the range of applicability of the standard lubrication theory.

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Challenges of mathematical and numerical modelling of the atmosphere dynamics for weather prediction

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Abstract. In this talk we will discuss, from a historical perspective, the evolution of the mathematical modelling of atmosphere dynamics and its application to weather forecasting. Modern approaches and the challenges of new generation of models will be presented with examples of the existing open problems on mathematical and numerical aspects.

Dispersion Relation Preserving Optimization for High Wavenumber Midpoint Interpolation

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Abstract. Optimization in the wavelength space is proposed to interpolations with central numeric stencils. Applications include primarily the area of Computational Fluid Mechanics (CFD) and other numerical methods based on spatial discretization. Maximum precision is ensured by Taylor series expansion where one of the degrees of freedom is then used to minimize error in the spectral space. Explicit and implicit forms are proposed for various orders of precision from the stencil numeric variation. Results show optimal gain in numerical error for high wave numbers by modifying the coefficients of the interpolation schemes, maintaining the numerically defined precision.

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MRI and Navier-Stokes Equations: How do they relate for a better exam?

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Abstract. Magnetic resonance imaging (MRI) is an important medical imaging modality used for diagnosis, disease prevention / monitoring and treatment planning. MRI produces excellent

quality anatomical and functional images, in particular blood flow images. Considering that 31% from the causes of death on adults in 2019 worldwide are related to heart disease, using MRI to diagnose and prevent cardiovascular diseases is in many cases necessary. However, its high costs plus prohibitive examination times can limit its use in the cardiovascular scenario, showing the need for acceleration. This talk aims to present how anatomical and blood flow images are acquired. As well as the image reconstruction techniques based on under-sampled acquired signals. Also in the context of under-sampling, show how the Navier-Stokes equations (NSE) can be used to assist the reconstruction process, and thus accelerate MRI examination. Finally, some computational and theoretical difficulties about the model based MRI acceleration problem will be briefly treated, as well as some benefits in addition to the acceleration.

Influence of non-Newtonian blood flow models on drug deposition in the arterial wall

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Abstract. In this talk, we investigate the influence of non-Newtonian blood flow models on drug diffusion from a coronary drug-eluting-stent (DES). We consider the Oldroyd-B, Phan-Thien-Tanner (PTT) and Giesekus viscoelastic models for the description of fluid dynamics of blood. The model for blood flow is coupled with plasma filtration and mass transport from a DES. The model for the transport problem takes into consideration non-Fickian diffusion, drug dissolution, polymer degradation and binding. We propose an Implicit-Explicit (IMEX) finite element method and show numerical experiments that confirm the effectiveness and order of convergence of the employed methodology.

Trapped Waves and Collisions

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Abstract. The aim of this talk is to discuss trapped waves solutions for the full Euler equations due to a localized pressure distribution along the free surface. We consider the case where the Froude number is greater than 1 and study collisions of trapped waves in a low pressure region with solitons. The collisions present rich features, such as destruction of the trapped area, switch of the wave which stays trapped and appearance of a wave that looks like a blocked wave. This is an ongoing work in collaboration with Prof. Marcelo V. Flamarion (UFRPE).

Multiscale domain decomposition methods for the subsurface flow simulation of oil recovery

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Abstract. The Multiscale Robin Coupled Method (MRCM) is a domain decomposition method that has been introduced to efficiently approximate velocity and pressure fields for single-phase flows in highly heterogeneous porous media (Guiraldello et al., 2018). Its discrete variational formulation generalizes other well-established multiscale domain decomposition mixed methods, such as the Multiscale Mortar Mixed Finite Element Method (MMMFEM) (Arbogast et al., 2000 & 2007), the Multiscale Hybrid Mixed Finite Element Method (MHM) (Harder et al., 2013), and the Multiscale

Mixed Method (MuMM) (Francisco et al., 2014). The MRCM rely on the imposition of a Robin-type boundary conditions on the interfaces between the non-overlapping subdomains, such that the solution of all these methods can be recovered by a suitable choice of the Robin boundary parameter and the approximation spaces for the interface unknowns, which are in fact the interface pressure and normal fluxes. The individual choices of discrete spaces for both these quantities introduces a great deal of flexibility to the method (Guiraldello et al., 2019). The MRCM was well explored for the solution of single-phase flows and has been recently extended for two-phase flows in porous media. By introducing an operator splitting strategy, the scalar conservation law for the saturation of one of the phases and the velocity field can be updated sequentially. In this scenario, the suitable choice of parameters and discrete interface spaces for the MRCM is thoroughly investigated. Results show unprecedented improvement in accuracy when compared to other multiscale mixed methods available in the literature, when simulating highly heterogeneous petroleum reservoirs.

Computational Simulation of Fluid Flow Magnetic in Cavities

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Abstract. In this work we studied flows of magnetic fluids in a lid driven cavity. We proposed a vorticity-streamfunction formulation to solve the flow and an evolution equation for the magnetization. The equations governing the flow are solved using a finite differences scheme. We identified the main dimensionless physical parameters of the problem. The magnetic field is generated by a conductive wire, is locally constant and the fluid is considered weakly magnetizable. With this well-known geometry we study the effects of each of the terms of the magnetization equation in the flow. In order to obtain flows with more intense distributions of velocities and vorticity throughout the cavity, we studied several configurations of moving walls in the cavity. It was observed that for high values of the magnetic pressure coefficient, flows in the absence of the precessional term, the steady state regime of the flow is periodic. In addition, the increase in the magnetic pressure coefficient leads us to aperiodic regimes. Simulations with homogeneous field were performed in an attempt to isolate the effects of the precession term on the magnetization equation and the magnetic torque term in the rotational of the Navier-Stokes equation.
This is an ongoing work jointly with Y. Sobral and F. Cunha.

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Posters

Computer simulation of colapsing columns of granular materials using the discrete element method.

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Abstract. We have implemented a computer simulation of collapsing columns of granular materials using the discrete element method. In this simulation, we consider only the normal contact and friction forces of type particle-particle and particle-wall. The normal contact forces follow the Hertzian model while the friction forces follow the Cundall-Strack model. Calling the height of a column as H and the length of the base as L, we can define the aspect ratio of the column as $a = H/L$. We have simulated columns with different aspect ratios and preliminary results suggest that the runout distance scales with a power of the aspect ratio. The simulation was implemented in C++ and the rendering uses the SFML library. It was written using TDD (Test Driven Development) and is covered by, approximately, 200 unit test written using the Google Test library.

Steady Flow of a Uniform Rivulet Down a Vertical Wall

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Universidade de Brasília

Abstract. We investigate the steady flow of a uniform rivulet down a vertical wall. This problem has many applications, which are easily found in everyday life, such as paint running down on a canvas and chocolate syrup flowing over a cake. In nature, examples can be seen in the lava flow from an erupting volcano and melting glaciers. Initial studies, such those of Towell & Rothfeld [1] and Allen & Biggin [2], employed a thin-film approximation to determine the free surface profile, as a function of the static contact angle, and the velocity field. Duffy & Moffatt [3] have also studied the problem within the framework of the lubrication theory. In the present study, we have mapped the physical domain (free-surface profile) onto a rectangular computational domain and then solved the full problem (Navier-Stokes equations) numerically using a finite difference method. The code was validated with an exact solution presented by Perazzo & Gratton [4]. Furthermore, a new lubrication approximation was formulated using gradient dynamics based on the recent work of Lopes et al. [5]. The results were compared with the ones obtained previously in the literature and it was found that the new developed model is not only the most accurate among the reduced models, but also that it works well beyond the range of applicability of the standard lubrication theory.

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Poiseuille Flow in a Duct of Rectangular Cross Section

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Abstract. Ducts are configurations that inflate or exhaust air throughout a building, thus negating the need to use fans installed in strategic locations. In fact, ducts are the most used way to transport air, promoting one of the most priority actions currently in a corporate space: air renewal to promote comfort in the workplace. A good air distribution and circulation design aims at an effective balancing of the system in order to maintain the optimum conditions of comfort and safety in the environment to be conditioned, together with low operating and maintenance costs. In the present study, we investigate a classical fluid dynamics problem: the Poiseuille flow in a duct of rectangular cross section. We solve this problem analytically using separation of variables and numerically using finite differences.

Transient Couette Flow Problem: Analytical and Numerical Solutions

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Abstract. Couette flow is a viscous flow between two parallel plates separated by some vertical distance. The lower plate moves with constant velocity and the upper plate is kept stationary. In the present work, the governing equations are solved analytically by the method of separation of variables (Fourier method) and numerically solved by employing the FTCS (Forward-Time Central-Space) method. Both solutions are presented and compared graphically.

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Number Theory

Talks

On the number of fully weighted zero-sum subsequences.

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Abstract. Let G be a finite additive abelian group with exponent n and $S = g_1 \cdots g_t$ be a sequence of elements in G. For any element g of G and $A \subseteq \{1, 2, \ldots, n-1\}$, let $N_{A,g}(S)$ denote the number of subsequences $T = \prod_{i\in I} g_i$ of S such that $\sum_{i\in I} a_i g_i = g$, where $I \subseteq \{1, \ldots, t\}$ and $a_i \in A$. We prove that $N_{A,0}(S) \geq 2^{|S|-D_A(G)+1}$, when $A = \{1, \ldots, n-1\}$, where $D_A(G)$ is the smallest positive integer l, such that every sequence S over G of length at least l has nonempty subsequence $T = \prod_{i\in I} g_i$ such that $\sum_{i\in I} a_i g_i = 0$, $I \subseteq \{1, \ldots, t\}$ and $a_i \in A$. Moreover, we classify the sequences such that $N_{A,0}(S) = 2^{|S|-D_A(G)+1}$, where the exponent of G is an odd number.

On Mahler's U_m-numbers

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Abstract. The genesis of transcendental number theory, took place in 1844 with Liouville's result on the "bad" approximation of algebraic numbers by rationals. More precisely, if α is an algebraic number of degree $n > 1$, then there exists a positive constant C, such that $|\alpha - p/q| > Cq^{-n}$, for all $p/q \in \mathbb{Q}^*$. Using this remarkable fact, he was able to build a non-enumerable set of transcendental numbers called *Liouville numbers*. Since then, several classifications of transcendental numbers have been developed, one of them proposed by Kurt Mahler in 1932. He splitted the set of transcendental numbers on three disjoint sets: $S-$,$T-$ and $U-$numbers. In a certain sense,$U-$numbers generalize the concept of Liouville numbers. Yet, the set of $U-$numbers can be splitted into U_m-numbers, that are numbers "rapidly" approximable by algebraic numbers of degree m.

On this lecture, the following result, made in cooperation with D. Marques, will be proved: Let $\omega : \mathbb{N} \to \mathbb{N}$, such that $\omega_n \to \infty$, as $n \to \infty$. Let $\xi \in \mathbb{R}$ be a Liouville number, such that there exists an infinite sequence of rational numbers $(p_n/q_n)_n$, satisfying

$$\left|\xi - \frac{p_n}{q_n}\right| < H\left(\frac{p_n}{q_n}\right)^{-\omega_n},$$

where $H(p_{n+1}/q_{n+1}) \leq H(p_n/q_n)^{O(\omega_n)}$. Now, take $\alpha_0, \ldots, \alpha_l, \beta_0, \ldots, \beta_r \in \overline{\mathbb{Q}}$, with $\beta_r = 1$ and $\alpha_l \neq 0$, such that $[\mathbb{Q}(\alpha_0, \ldots, \alpha_l, \beta_0, \ldots, \beta_r) : \mathbb{Q}] = m$. Then, for $P(z), Q(z) \in \overline{\mathbb{Q}}[z]$, given by $P(z) = \alpha_0 + \alpha_1 z + \cdots + \alpha_l z^l$ and $Q(z) = \beta_0 + \beta_1 z + \cdots + \beta_r z^r$, $P(\xi)/Q(\xi)$ is a U_m-number.

Gröbner Basis methods in Projective Reed-Muller codes

Victor Gonzalo Lopez Neumann (victor.neumann@ufu.br)
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Abstract. Projective Reed-Muller codes were introduced by Lachaud in 1988 and their dimension and minimum distance were determined by Serre and Sørensen in 1991. In coding theory, one is also interested in the higher Hamming weights, to study the code performance. In this talk we present several values for the next-to-minimal weights of projective Reed-Muller codes using techniques from Gröbner basis theory. This is a joint work with Cícero Carvalho.

Exact values of the function $\Gamma^*(k)$

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Universidade Federal de Goiás

Abstract. The function $\Gamma * (k)$ is defined to be the smallest number s such that a diagonal form of degree k in s variables with (rational) integer coefficients is guaranteed to have a nontrivial zero in the p-adic fields for all primes p. In this work, we find the exact value of $\Gamma * (k)$ many values of $k \leq 64$.

A tiling approach to integer partitions

José Plínio de Oliveira (josepli@ime.unicamp.br)
Universidade Estadual de Campinas

Abstract. In this paper we present combinatorial interpretations for unrestricted partitions in terms of tiling. This same procedure allows interpretation for several other important constrained partition sets.

Pair of primitive elements over finite fields

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Universidade Federal de Uberlândia

Abstract. In this talk we present conditions for the existence of primitive element z in a finite field F_q and a rational function $f(x)$ such that $f(z)$ is also primitive in F_q.

Generalized Weierstrass semigroups and their Poincare series

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Universidade Estadual de Campinas

Abstract. This is a joint work with J.J. Moyano-Fernandez and W. Tenorio. We investigate the structure of the generalized Weierstrass semigroups at several points on a curve defined over a finite field. This allows us to associate to certain monoids of $\mathbb{Z}^t$ combinatorial invariants such as the Poincare series and the semigroup polynomial. As a matter of fact, the semigroup polynomial determines the Poincare series and this series determines completely the monoid. Finally, we discuss functional equations under symmetry assumptions.

Generalized Weierstrass semigroups of certain curves with separated variables

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Universidade Federal de Goiás

Abstract. In this talk we present some results concerning the structure of the generalized Weierstrass semigroups at several points satisfying special conditions. (Joint work with G. Tizziotti).

Probability

Talks

Exact pathwise simulation of multi-dimensional Ornstein-Uhlenbeck processes

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Escola de Matemática Aplicada - Fundação Getúlio Vargas

Abstract. The exact scenario simulation of multidimensional processes of the Ornstein-Uhlenbeck type is considered. We propose new methods that allow the exact simulation of this process and, simultaneously, the generation of the underlying Wiener trajectories from the same source of randomness. This is particularly important when both processes are system-components in larger stochastic models, for which the study of pathwise dynamics is required.

Full Partition Markov Models

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Abstract. In this paper, we analyze the model proposed in [1] in which is considered a set of p independent samples of discrete time Markov chains, over a finite alphabet A and with finite order o. The model is obtained identifying the states on the state space A^o where two or more samples share the same transition probabilities (see also [2]). This identification establishes a partition on $\{1, ..., p\} \times A^o$ the set of samples and the state space. We show that by means of the Bayesian Information Criterion (BIC) the partition can be estimated eventually almost surely. Also in [1] is given a notion of divergence, derived from the BIC, which serves to identify the proximity/discrepancy between elements of $\{1, ..., p\} \times A^o$ (see also [3]). In the present article, we also prove that this notion is a metric in the space where the model is built and that it is statistically consistent to determine proximity/discrepancy between the elements of the space $\{1, ..., p\} \times A^o$. We apply the notions discussed here for the construction of a parsimonious model that represents the common stochastic structure of 153 complete genomic Zika sequences, coming from tropical and subtropical regions.

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Joint work with: M. T. A. Cordeiro, Jesús E. García and S. L. M. Londoño.

The first passage problem for stable linear delay equations perturbed by small power law Lévy noise

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Abstract. In this talk we present a linear scalar delay differential equation subject to small multiplicative power tail Lévy noise. We solve the first passage (the Kramers) problem with probabilistic methods and discover an asymptotic loss of memory in this non-Markovian system. Furthermore, the mean exit time increases as the power of the small noise amplitude, whereas the pre-factor accounts for memory effects. In particular, we discover a non-linear delay-induced exit acceleration due to a non-normal growth phenomenon. Our results are illustrated for the linear delay oscillator driven by alpha-stable Lévy flights.

Existence of solution for equations with stochastic representation.

Diego Sebastián Ledesma (dledesma@ime.unicamp.br)
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Abstract. We will study the existence of a solution to a particular type of equation that admits a stochastic representation.

Fractionally Integrated Moving Average Processes with Long-Range Dependence: Estimation and Asymptotic Properties

Sílvia Regina Costa Lopes (silviarc.lopes@gmail.com)
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Abstract. We construct a moving average stochastic process, whose kernel coincides with the one derived from the Generalized Ornstein-Uhlenbeck Type Process (GOUT), but considering the noise process as a fractional Brownian motion process. From the Riemann-Liouville fractional integrals theory, we propose a fractionally integrated moving average process, for which we highlight some results, prove its long-range dependence property, present na estimator for the long-range dependence parameter and show its asymptotic properties.

Haar systems, groupoides, quasi-invariant probabilities and Thermodynamic Formalism

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Abstract. We will present the concepts of transverse function, Haar systems, multiplicative cocycle, quasi-invariant probabilities and relate all this to classical problems in Thermodynamic Formalism and to the Ruelle operator.

Strong-Trajectorial approximation $2D$ -Navier Stokes by stochastic particle systems with moderate interaction

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Abstract. We consider an interacting particle system modeled as a system of N stochastic differential equations driven by Brownian motions with moderate interaction. We prove that the empirical process (mollifiers) converges, uniformly in the space variable, to the solution of the 2D-Navier Stokes Equation written in vorticity. We justify the convergence of the numerical method proposed by Chroin for the case the particle systems with moderate iterations.

The geometric structure of probability spaces

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Abstract. In this talk we shaw discuss some geometric properties of probability spaces. Then we will see how these structures can be used to infer ergodic properties of dynamical systems.

a diffusion. By the techniques of Hausmann and Pardoux that process solves one SDE involving its marginal distributions. This constitutes an interesting special McKean SDE for which (under some technical assumpions) well-posedness can be studied.

The Constrained-degree percolation model

Remy de Paiva Sanchis (rsanchis2@gmail.com)
Universidade Federal de Minas Gerais

Abstract. Let k be a positive integer and consider a sequence of iid uniform random variables on [0.1] indexed by the edges e of a graph G, denoted by Ue. In the Constrained-degree percolation model each bond is closed at $t = 0$ and tries to become open at time $t = Ue$, it succeeds if both its end-vertices have degrees at most $k - 1$ by that time.

We prove a non-trivial phase transition theorem for this model on the 2D square lattice, as well on the d-ary regular tree. W e also prove that on the square lattice the infinite cluster is unique in the supercritical phase.

Joint work with B.N.B. Lima, D.C dos Santos, R. Teodoro and V. Sidoravicius.

Functional Itô Calculus and Applications to Stochastic Control

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Fundação Getúlio Vargas

Abstract. In this talk we will review the recent developments on Functional Itô calculus, FITO in short. Created (or discovered) by Bruno Dupire and published in a seminal paper in 2009, this calculus is a generalization of Itô's classical theory and allows us to examine models where the history of certain factors plays an important role. We will present the general theory and survey the theoretical unfolding of FITO. As an application, we will show how this theory allows us to consider stochastic control problems with path-dependence influence of the control in the dynamics of the state process.

On the Martin boundary for hyperbolic groups

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Abstract. In this talk, I present recent progress in the geometrization of the Martin boundary of random walk on a hyperbolic group with psi-mixing increments. This setting is equivalent to consider a random walk driven by an expanding dynamical system and allows to identify the Martin boundary with a certain subset of sigma-finite conformal measures, which is a well-known object in the theory of dynamical systems, but less known in probability theory. Moreover, as the group is hyperbolic, it is possible to identify the Martin boundary with the visual boundary of the group.

Posters

Analysis of relative air humidity in Brasília - DF: an application of variable order chains

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Abstract. Following Quintino and Moreira (2015), the initial motivation of this work was to obtain meteorological models in the Federal District. More specifically, we propose models of prediction for the minimum relative humidity of air for Brasília-DF. For this, the class of models used were the variable order chains. We use daily minimum relative air humidity data to calculate the probability that a state of emergency, alertness, attention state, or adequate humidity for health will occur on a given day. The estimation of the trees of contexts associated to the proposed models was done through the BIC estimator in the R environment. In addition, we verified that there was a change in the relative air unit of Brasilia after 2000 years when compared to previous years.

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Mallows Distance as a Mode of Convergence in the Asymptotic Behavior of a Classic Stochastic Process: Recent Results and Statistical Simulations

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Instituto Federal de Brasília

Abstract. The present study addresses the characterization of Mallows diatance convergence of the Empirical Process generated by random samples. The theme guides a Scientific Initiation promoted by FAP / DF. The objective is to understand the relationship of the Empirical process with the Gaussian Distribution through Mallows Distance, in particular, order 2 and other orders, providing statistical simulations of these cases. The scientific community will be detailed about the research results and the scope of their motivating problem. Additionally, we will present the elementary results modeling of probability theory by computational implementation. Given a random sample $X_1, X_2, \ldots, X_n$ and its distribution F, by the Law of Great Numbers and the Glivenko-Cantelli Theorem, F can be well estimated by the constructed empirical distribution function F_n from the data. Historically, this fact has given relevance to the study of the stochastic process called the empirical process, in addition, the study of convergences was fostered for the understanding of the asymptotic behavior of data obtained in real situations. Given the empirical process $\sqrt{n}(F_n(x) - F(x)), x \in \mathbb{R}$ associated with $X_1, \ldots, X_n$ of F, we have a relationship with the Gaussian distribution via Central Limit Theorem. Thus, the study of the asymptotic behaviour of empirical process refers to the so called goodness of fit tests, which are routines aimed to try the distributional identity from a given sample. A recent approach to goodness of fit tests is based on the Mallows distance metric. Some Mallows distance applications to Statistics address the relation of this metric and partial sums, according to [3], [1], [2] e [4]. The convergence of empirical process β_n in the metric modality; that is, the asymptotic behaviour of $d_\alpha(\beta_n, Z)$, is a theme of interest because this metric modality is stronger than the classic ones and more widespread lately in statistical simulations, [4] e [3]. The theme itself is also a great student initiation in the modern world of research in Probability and Statistics, as it aggregates a lot of Probability theory concepts and classic facts with two of the latest research trends in production of knowledge in said field, namely: the approach of distributions, which are asymptotic limits to partial sums (called stable distribution), and Mallows distance as a convergence metric.

This work was carried out under the guidance of Prof. Dr. Wembesom Mendes Soares (Instituto Federal de Brasília).

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Stable distributions and the ruin theory

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Abstract. Gaussian distributions and processes are studied in various applications and their utility in stochastic models is well accepted. However, finance and insurance analyzes often indicate the presence of heavy tails, so stable processes prove to be an appropriate resource for heavy tail probabilistic models. Accordingly, the research deals with the classic risk model for insurance activity, which is part of risk theory, one of the branches of actuarial mathematics.

The theme guides a scientific initiation process that explores the treatment in the theoretical model of the temporal evolution of the capital of a continuous insurer, essentially the so-called classical model that is based on a renewal process, whose study began considering Poisson distribution and ends considering stable distributions, thus the theory of stable distributions and α-stable Lévy processes will be explored.

The central interest is to study uncertainty and to analyze the processes that define the paths taken by insurers to fulfill their obligations, it means the moments the company needs to pay any of the claims.

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Model of wind behavior in the city of Brasília using Variable Order Chains

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Abstract. In this work we used Variable Length Chains to propose a statistic model for the wind's behavior in the city of Brasília. These models where originally presented on Rissanen (1983), who called context the porcion of the past necessary to predict the next symbol. As no context is a propper sufix of another context, we can represent the set of all contexts from a variable lenght process through a probabilistic tree. In this point we suppouse that the wind's intensity is a variable length process which takes the value 1 if there is strong wind and takes the value 0 otherwise. Initially, we considered daily mesures of the wind's velocity at Brasília, which are associated with the period between 08/21/1961 and 07/15/2019, to obtain the predict models of the wind's intensity. In addiction we have shown that has been a change in the wind's behavior at Brasilia after the 2000s.

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Theory of Computation

Talks

Formalizing Termination of Functional Programs and Term Rewriting Systems in PVS

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Abstract. Termination is an important property for both functional programs and term rewriting systems. Although undecidable, several semi-decision strategies have been used to address such problem and reason over it, such as the Size Change Principle [2], Type Check Conditions, Calling Context Graphs [3] and Dependency Pairs [1]. Formalizations for the correctness of such termination criteria have been provided in several proof assistants in order to improve automation of termination of given specifications. Such formalizations in the Prototype Verification System (PVS) [4], a proof assistant with a functional specification language that allows higher order logic and performs proofs following the Gentzen Calculus of Sequents, will be discussed, including how criteria in both functional programs and term rewriting systems can be related and used for increase automation level of termination proofs.

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Properties of Rule-Based Anti-Unification Algorithms

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Abstract The anti-unification problem is concerned with finding an expression that generalizes two input examples. Interesting generalizations are the least general ones. They inherit as many common features of the input examples as possible and uniformly represent their differences by variables. Such generalization problems arise in many areas of computer science and mathematics. An algorithm for constructing a least general generalization of two given input examples is called an anti-unification algorithm. In this talk we will discuss some properties of rule-based anti-unification algorithm that allow us to use them for solving the matching problems.

β-equivalence of linear planar λ-terms

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Abstract. The correspondence between linear λ-terms and trivalent rooted maps inherits some interesting properties from rooted maps to linear λ-terms. As an example, the genus is one of them. Using the genus of a λ-term to measure its complexity, the λ-term with genus zero (or planar terms) are the simplest. And for this kind of λ-terms there are easier procedures to decide β-equivalence.

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Bounded-Degree Fixed-Points in Linear Time

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Abstract Solutions to specific problems can be generalized to entire classes of problems defined by logical languages. This is the case of Courcelle's Theorem, which states that any problem of graphs which can be expressed in the language of monadic second-order logic can be decided in linear time on graphs of bounded tree-width.

Another result in the same spirit of Courcelle's Theorem is Seese's Theorem, which states that any property of graphs which can be expressed in first-order logic can be verified in linear time on graphs of bounded degree.

Lindell extended Seese's Theorem to the problem of computing certain sets of vertices inductivelly defined. Such sets can be defined as the least fixed-point of monadic operators on graphs of bounded degree. Lindell shows that such least fixed-points can be computed in linear time using a random access machine (RAM).

We show how to extend Lindell's result to the problem of computing sets of vertex pairs inductivelly defined, provided that these sets have bounded degree as well. We show that these sets can also be computed in linear time using a RAM.

Principal Typing for the $\lambda\sigma_{dB}$-calculus

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Abstract. The $\lambda\sigma_{dB}$-calculus is inspired on $\lambda\sigma$-calculus proposed in [1]. It is a calculus of explicit substitutions (CES) containing all De Bruijn indexes in its syntax. The CES's are extensions of the

λ-calculus that implement concretely its main operation, which is the β-reduction since they contain the operation of substitutions as part of their language. The $\lambda\sigma_{dB}$-calculus is confluent (ground) and simulates the classical β-reduction. A version of $\lambda\sigma_{dB}$ with simple types was considered, which we called $B1$-system. The $B1$-system enjoys properties as subject reduction and soundness, and, in Church-style, $B1$ satisfies the type-uniqueness property. Terms may have many types depending on the context in Curry-style. Therefore, another important property to study is the *principal typing* (PT for short), which has also been called *principal pair* in [2]. The PT property in a type system responds to question whether for a given term its most general typing can be found [3]. A system-independent definition of PT due to Wells was proposed in [4]. He has proved that it generalizes previous system-specific definition. In this work, a new definition of PT concerning B1 is proposed and it is proven to be equivalent to that Wells' definition. Finally, we prove PT property for $B1$ according to the proposed definition from a type inference algorithm.

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Formalization of Rice's Theorem for a Turing Complete Functional Language Model

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Abstract. Classical proofs of Rice's Theorem assume the existence of a universal (Turing) machine and build a reduction from the problem of deciding whether a machine halts or not to the problem of separability of semantic properties of machines. This work presents a formalization in PVS of Rice's Theorem for a computational model given as a class of partial recursive functions. The model is build over basic operators that when restricted to successor, projections, greater than and bijection from tuples of naturals to naturals, results in a model that is formalized to be Turing complete. The main differences with classical proofs are that the given formalization is developed for a functional programming model and that the proof does not depend on the undecidability of the Halting Problem, being made directly without using any translation to or from other computational models. As corollaries, straightforward formalizations of the undecidability of the Halting Problem, functional equivalence problem, existence of fixed points problem and self-replication problem are obtained.

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Extending the Locally Nameless Representation with an Explicit Substitution Operator

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Abstract. The Locally Nameless Representation[1] is a formal framework developed in the Coq proof assistant [3] for formalizing languages with binders. In this framework, bound variables are represented by DeBruijn indexes, while free variables are represented by names. This way, terms have a unique representation modulo α-conversion, therefore equivalence between terms corresponds to syntactic equality. In this work, we show the current development of an extension of Charguéraud's framework with an explicit operator for the substitution operation. The expressions are built inductively by the grammar of pre-terms, a super set of terms. In particular, we provide three different, but equivalent, characterizations for terms and several auxiliary lemmas that will be useful in a formalization of an extension of the λ-calculus with explicit substitutions [2].

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Nominal C-unification

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Abstract. The nominal approach allows us to extend first-order syntax and represent smoothly systems with variable bindings. Nominal unification is, therefore, the extension of first-order unification modulo-equivalence by taking into account this nominal setting. In this talk, we revisit nominal

C-unification (nominal unification with commutative operators) and talk about our current work in progress in this area. This is a joint work with Mauricio Ayala-Rincón, Daniele Nantes-Sobrinho, Maribel Fernández and Washington de Carvalho-Segundo.

Reductions between certain incidence problems and the Continuum Hypothesis

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Abstract In this work, we consider two families of incidence problems, $\mathcal{C}_1$ and $\mathcal{C}_2$, which are related to real numbers and countable subsets of the real line. In both cases, to solve the problem one has to give an appropriate (but random[4]) response to a certain initial data; notice that such procedure could also be interpreted as some one-round game between two players, were the first player gives the initial data and the second player wins if he gives a response which solves the problem. Problems of $\mathcal{C}_1$ are as follows: given a real number x, pick randomly a countably infinite set of reals A hoping that $x \in A$, whereas problems in $\mathcal{C}_2$ are as follows: given a countably infinite set of reals A, pick randomly a real number x hoping that $x \notin A$. One could arguably defend that, at least intuitively, problems of $\mathcal{C}_2$ are easier to solve than problems of $\mathcal{C}_1$. Our main results are the following:

1. After some suitable formalization, we prove (within **ZFC**) that, on one hand, problems of $\mathcal{C}_2$ are, indeed, at least as easy to solve as problems of $\mathcal{C}_1$.

2. On the other hand, the statement "Problems of $\mathcal{C}_1$ have the exact same complexity of problems of $\mathcal{C}_2$" is shown to be an equivalent of the Continuum Hypothesis (**CH**).

The suitable formalization for the notion of comparison of complexities between problems will be given by *reductions*: for instance, problems in $\mathcal{C}_2$ will be shown to be *simpler* (or *not more complicated*) than the ones in $\mathcal{C}_1$ because we give a **ZFC** proof that the (act of) solving a problem in $\mathcal{C}_2$ may be reduced to solving a (corresponding) problem in $\mathcal{C}_1$. Those reductions will be given in terms of morphisms between objects of the category $\mathcal{PV}$, which is a subcategory of the dual of the simplest case of the Dialectica Categories introduced by Valeria de Paiva ([4,5]); such morphisms are also known as *Galois-Tukey connections*, which is a terminology due to Peter Vojtáš ([9]). Several connections between this category and Set Theory have been extensively studied by Andreas Blass in the 90's (see e.g. [1,2]), and, more recently, have also been investigated by the author ([7,8]). A full description of how the morphisms of $\mathcal{PV}$ correspond to reductions between certain problems may be found at the pages 62 and 63 of [1].

And, as randomly taken countable sets of real numbers may be regarded (in a certain oriented thought experiment) as *the set of punctures of a countable set of darts thrown at the real line*, the proof of our announced equivalence for the Continuum Hypothesis is, in fact, pretty similar to the one presented by Freiling in [3] – whose mathematical content, however, is due to Sierpiński, in his classical monograph on **CH** ([6]). However, in our context Freiling's controversial assumption of symmetry seems unnecessary, and our approach lead us, apparently, to an even more dramatic discussion – if one considers the following question:

[4]This paper deals with some problems which may be viewed as *thought experiments*. In order to state such thought experiments, we have chosen to proceed with a certain *ad hoc* identification, mostly by stylistic reasons: we will identify *randomness* with *arbitrariness*. In practice, such identification consists in the assumption that, when we declare that we are picking arbitrary objects, then there is no pattern involved and all possible outcomes are unpredictable since they obey an equal probability distribution.

- Before being given a countable set A of reals and a real number x, both to be randomly taken, should one say that it will be *easier* (or it will be *more likely*) that, eventually , this real number x *will miss* the countable set A? Or should one say that, under the very same conditions and interpretations, *it will hit it*?

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A logical framework for exogenous component-based software reasoning with Reo

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Abstract Software components communicate using channels for message passing. Reo is a graphic-based coordination modelling language which aims to capture and model this interaction. The fact that Reo can be used to model many real-world situations has attracted attention from researchers, resulting in a great effort tailored to formalize its behaviour in order to verify properties of Reo circuits. We present a logical framework giving rise to ReoXplore, an open-source tool to model and reason about Reo circuits in a proof-theoretical way (using Coq proof-assistant) and by means of model checking them (using nuXmv model checker).

Minicourse

Interactively Proving Mathematical Theorems

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Abstract. Although the acceptance of a mathematical truth will always depend on humans, nowadays finding researchers working together interactively with machines to produce formal proofs is no longer seen as an innovation but a requirement of modern mathematics (and mathematicians well-practice). This short-course will survey "proof theory" and "logical deduction" and, using the proof assistant PVS, participants (students and researchers at any level) will "put their hands in the dough" receiving a basic training to understand how to proof theorems using such powerful tools.

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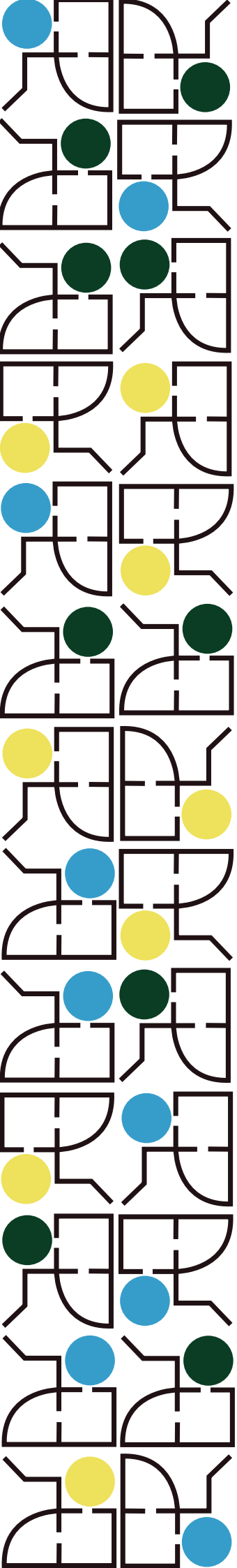

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