# EXERCISES <br> "AUTOMATA, LANGUAGES, AND GROUPS OF AUTOMORPHISMS OF ROOTED TREES" 

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Exercise 1. Let $A$ be a nonempty finite alphabet. Is $A^{*}$ countable? Why?

Exercise 2. Consider the following automaton, with intial state $q_{0}$, final state $q_{1}$, and transition function:

|  | a | b |
| :---: | :---: | :---: |
| $q_{0}$ | $q_{0}$ | $q_{1}$ |
| $q_{1}$ | $q_{0}$ | $q_{2}$ |
| $q_{2}$ | $q_{2}$ | $q_{1}$ |

Are the strings $a^{3} b, a b a a b, a b a b a b a b, a^{4} b b a, a^{10} b^{8} a b, a^{6} b^{8} a b$ accepted?

Exercise 3. Determine the language accepted by the automaton with initial state $q_{0}$, final state $q_{1}$, and transition function:

|  | 0 | 1 |
| :---: | :---: | :---: |
| $q_{0}$ | $q_{1}$ | $q_{2}$ |
| $q_{1}$ | $q_{1}$ | $q_{1}$ |
| $q_{2}$ | $q_{1}$ | $q_{0}$ |

Exercise 4. Let $L=\left\{w \in\{a, b\}^{*}| | w \mid \equiv 1 \bmod 3\right\}$. Define a DFA $A$ such that $L=L(A)$.

Exercise 5. Construct a NFDA that accepts the language $L=\{x \in$ $\{0,1,2\}^{*}| | x \mid \geq 5$ and the 4 th symbol from the right is 0$\}$.

Exercise 6. Determine if the following languages are regular or not:

- $L=\left\{x \in\{0,1\}^{*} \mid x\right.$ is palindrome $\}$
- $L=\left\{0^{i} 1^{j} \in 0,1^{*} \mid i, j>0,2 i>j\right\}$
- $L=\left\{x \in\{a, b\}^{*} \mid x\right.$ starts with $a$ and ends with $\left.a b\right\}$.

Hint: Pumping Lemma;

Exercise 7 ( $^{*}$ ). Find a DFA equivalent to the following NDFA:


Do you remember Context-Free languages?
Exercise 8. Which of the following is a context-free language?

- $L=\left\{a^{n} b^{m} a^{m} b^{n} \mid n, m \geq 0\right\}$
- $L=\left\{a^{n} b^{n} a^{m} b^{m} \mid n, m \geq 0\right\}$
- $L=\left\{a^{n} b^{m} a^{n} b^{m} \mid n, m \geq 0\right\}$
- $L=\left\{a^{n} b^{m} c^{q} \mid n=m\right.$ or $\left.m=q\right\}$.

