Universidade de Brasília
Instituto de Ciências Exatas
Departamento de Matemática

X Workshop on Nonlinear Differential Equations

September 04-08, 2017

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Universidade de Brasília

X Workshop on Nonlinear Differential Equations

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70910-900 Brasília - DF

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Paolo Piccione (USP, Brazil)
Vieri Benci (Università di Pisa, Italy)

About the meeting

This workshop is the tenth edition of a series of meetings which started in Brazil in 1996, originated mostly from the scientific collaboration between Italy and Brazil. Since then, the meeting has been held almost every two years: the most recent workshops took place in Rio-Brazil (2008), Verbania-Italy (2010), Joo Pessoa-Brazil (2012) and Varese-Italy (2015). In all of them, we had the participation of mathematicians not only from Italian and Brazilian institutions, but also from many other countries in Europe, U.S.A. and Latin America.

Abstracts of the talks

Existence of heteroclinic solution for a double well potential equation in an infinite cylinder of $\mathbb{R}^N$

Claudianor O. Alves (coalves@mat.ufcg.edu.brm)
Universidade Federal de Campina Grande - UFCG, Brazil

Abstract. In this paper, by using variational methods we prove the existence of heteroclinic solutions for the following class of elliptic equations

$$-\Delta u + A(\epsilon x, y)V'(u) = 0 \quad \text{in} \quad \Omega,$$

where $\epsilon > 0$, $\Omega = \mathbb{R} \times \mathcal{D}$ is an infinite cylinder of $\mathbb{R}^N$ with $N \geq 2$. Here, we have considered a large class of potential V which includes the Ginzburg-Landau potential $V(t) = (t^2 - 1)^2$ and two geometric conditions on the function A. The first condition is the case where A is asymptotic at infinity to a periodic function, and the second one is when A satisfies

$$0 < A_0 = A(0, y) = \inf_{(x,y) \in \Omega} A(x, y) < \liminf_{|(x,y)| \to +\infty} A(x, y) = A_\infty < \infty, \quad \forall y \in \mathcal{D}.$$

On a Poincaré type inequality and the Sternberg-Zumbrun's conjecture

Ezequiel Barbosa (ezequiel@mat.ufmg.br)
Universidade Federal de Minas Gerais, Brazil

Abstract. In this talk we will discuss the application of a Poincaré type inequality to study local minimisers of perimeter with respect to volume within the Euclidean unit ball. Also, we will present a solution for the Sternberg-Zumbrun's conjecture.

Variational Methods for Singular Problems

Vieri Benci (vieri.benci@unipi.it)
University of Pisa

Abstract. We are interested in singular problems of the following type: find a function $u : \Omega \to \mathbb{R}$, which satisfies the following equation:

$$-\Delta u + u^2 - \frac{1}{u^3} = 0 \ \ in\ \Omega \tag{1}$$

with the following boundary condition:

$$u = g(x) \ \ for\ x \in \partial\Omega$$

where Ω is a bounded open set and $g \in L^1(\partial\Omega)$ is a function different from 0 which change sign; e.g. $g(x) = \pm 1$.

Formally, equation (1) is the Euler-Lagrange equation relative to the energy

$$E(u) = \frac{1}{2}\int \left(|\nabla u|^2 + u^2 + \frac{1}{u^2} \right) dx \tag{2}$$

and it might represent a model of phase transition where the enegy density assumes an infinite value when $u = 0$.

Clearly the energy (2) makes no sense for a function $u \in C^1(\overline{\Omega})$ and diverges for any function $u \in H_g^1(\Omega)$. Therefore it becomes difficult to interpret the meaning of equation (1); in particular is not clear which conditions to impose to the function u in the phase transition layer:

$$\Xi = \{x \in \Omega \mid u(x) = 0\}.$$

In this talk we analyse equation (1) in the framework of the theory of ultrafunctions. The ultrafunctions are a kind of generalized functions which have been recently introduced. They provide generalized solutions to equations which do not have any solution, not even among the distributions. The peculiarity of ultrafunctions is that they are based on the hyperreal field $\mathbb{R}^*$, namely the numerical field on which nonstandard analysis is based. The use of a non-Archimedean field, namely a field which contains infinite numbers, allows to treat energies as (2) in a consistent way and to give a meaning to equation (1).

Some results about critical points (minima, saddle points) of some integral functionals

Lucio Boccardo (boccardo@mat.uniroma1.it)
Sapienza, Università di Roma

Abstract. Some existence and regularity results about critical points (minima, crater points, saddle points) of some integral functionals will be presented; special emphasis will be given to the regularizing effect: that is, existence of solutions with finite energy, under assumptions that, a priori, do not give finite energy solutions.

The saddle points concern integral functionals defined in $(W_0^{1,2}, W_0^{1,2})$, so that the Euler-Lagrange equations are nonlinear elliptic systems.

$$\begin{cases} u \in W_0^{1,2}(\Omega): \ -\mathrm{div}(M(x)\nabla u) + A\,\varphi|u|^{r-2}u = f(x), \\ \varphi \in W_0^{1,2}(\Omega): \ -\mathrm{div}(M(x)\nabla\varphi) = |u|^r, \end{cases} \tag{3}$$

$$\begin{cases} u \in W_0^{1,2}(\Omega) \ : \ -\mathrm{div}((A+\varphi)\nabla u) = f(x), \\ \varphi \in W_0^{1,2}(\Omega) : \ -\mathrm{div}(M(x)\nabla\varphi) = \frac{1}{2}|\nabla u|^2, \end{cases} \tag{4}$$

where $r > 1$, $A > 0$, Ω is a bounded, open subset of R^N, with $N > 2$, f belongs to $L^m(\Omega)$, with $m \geq \frac{2N}{N+2}$, and $M(x)$ is a symmetric measurable matrix such that $M(x)\,\xi\,\xi \geq \alpha|\xi|^2$, $|M(x)| \leq \beta$, for almost every x in Ω, and for every ξ in R^N, with $0 < \alpha \leq \beta$.

Joint works with Luigi Orsina (Roma), Benedetta Pellacci (Napoli).

References

[1] L. Boccardo, B. Pellacci: Critical points of non-regular integral functionals; Revista Matemática Iberoamericana, to appear.

[2] L. Boccardo, L. Orsina: Regularizing effect for a system of Schrödinger-Maxwell equations; Adv. Calc. Var. 2016.

[3] L. Boccardo, L. Orsina: Existence via regularity of solutions for elliptic systems and saddle points of functionals of the Calculus of variations; Adv. Nonlinear Analysis, to appear.

Fractional order Orlicz-Sobolev spaces and applications

Julián Fernández Bonder (jfbonder@dm.uba.ar)
University of Buenos Aires, Argentina

Abstract. In this talk I will give what we believe is the *natural* definition of fractional order Orlicz-Sobolev spaces. After showing that this spaces are well-behaved from a functional analysis point of view, we investigate the limit as the fractional parameter $s \uparrow 1$. Then we apply these results to the study of some fractional non-standard growth elliptic-type problems.

Joint work with Ariel Salort (University of Buenos Aires).

Stability of ground state and renormalized solutions to mixed dispersion Schrdinger equations

Denis Bonheure (denis.bonheure@ulb.ac.be)
Université libre de Bruxelles

Abstract.
In this talk, we will be interested in standing wave solutions to a fourth order nonlinear Schrdinger equation having second and fourth order dispersion terms. This kind of equation naturally appears in nonlinear optics. First, we show existence of ground-state and renormalized solutions. We then discuss their qualitative properties, in particular their stability.

Based on joint works with Jean-baptiste Casteras, Ederson Moreira Dos Santos, Tianxiang Gou, Louis Jeanjean and Robson Nascimento.

Global folds between Banach spaces (as perturbations)

Marta Calanchi (marta.calanchi@unimi.it)
Università degli Studi di Milano, Italy

Abstract. Global folds between Banach spaces are obtained from a simple geometric construction: a Fredholm operator T of index zero with one dimensional kernel is perturbed by a compatible nonlinear term P. The scheme encapsulates most of the known examples and suggests new ones. Concrete examples rely on the positivity of an eigenfunction. For the standard Nemitskii case $P(u) = f(u)$ (but P might be nonlocal, non-variational), T might be the Laplacian with different boundary conditions, as in the Ambrosetti-Prodi theorem, or the Schrödinger operators associated with the quantum harmonic oscillator or the Hydrogen atom, a spectral fractional Laplacian, a (nonsymmetric) Markov operator. For self-adjoint operators, we use results on the nondegeneracy of the ground state. On Banach spaces, a similar role is played by a recent extension by Zhang of the Krein-Rutman theorem.

Joint work with Carlos Tomei and André Zaccur.

Nonlocal Schrödinger equations with H-L-S critical exponents

Daniele Cassani (daniele.cassani@uninsubria.it)
Università degli Studi dell'Insubria, Italy

Abstract. We are concerned with existence and concentration of ground state solutions to nonlocal Schrödinger equations, sometimes called Choquard-type equations. We consider the case in which the nonlinearity exhibits critical growth, namely in presence of the Hardy-Littlewood-Sobolev lower, as well as upper critical exponents. Assuming mild conditions on the nonlinearity and by using variational methods, we obtain existence and non-existence results. The case of dimension two is also studied.

Joint work with J. Zhang; J. van Schaftingen and J. Zhang; C. Tarsi and M. Yang.

Existence of a positive solution to a nonlinear scalar field equation with zero mass at infinity

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Universidad Nacional Autónoma de México, Mexico

Abstract. We establish the existence of a positive solution to the problem

$$-\Delta u + V(x)u = f(u), \qquad u \in D^{1,2}(\mathbb{R}^N),$$

for $N \geq 3$, when the nonlinearity f is subcritical at infinity and supercritical near the origin, and the potential V vanishes at infinity. Our result includes situations in which the problem does not have a ground state. Then, under a suitable decay assumption on the potential, we show that the problem has a positive bound state.

Joint work with Liliane Maia.

$\mathcal{D}^{1,2}(\mathbb{R}^N)$ versus $C(\mathbb{R}^N)$ Local Minimizer on Manifolds and Multiple Solutions for Zero-Mass Equations in $\mathbb{R}^N$

David G. Costa (costa@unlv.nevada.edu)
University of Nevada Las Vegas, USA

Abstract. We consider the question of existence of multiple solutions for a class of superlinear elliptic problems in $\mathbb{R}^N$ of the form

$$-\Delta u = b(x)g(u)\,,\ x \in \mathbb{R}^N\ \ (N \geq 3)\,,$$

where $g : \mathbb{R}^N \to \mathbb{R}$ is a continuous, subcritical nonlinearity, which is superlinear at $s = 0$ and at ∞, and natural hypotheses are imposed on the weight function $b(x)$ to render the above problem well-defined as a variational problem in the Hilbert space $H := \mathcal{D}^{1,2}(\mathbb{R}^N)$, the completion of $C_0^\infty(\mathbb{R}^N)$ under the norm $\|u\| = (\int |\nabla u|^2)^{1/2}$. Since $g'(0) = 0$, such problems are known in the literature as belonging to the zero-mass case of scalar field equations.

A main tool in our approach is the consideration of the subspace of H defined by

$$V := \{\, v \in H : v \in C(\mathbb{R}^N)\ \text{ with }\ \sup(1 + |x|^{N-2})|v(x)| < \infty\,\}.$$

Our main results are as follows.

First, we consider the corresponding functional to our problem

$$J(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2\ - \int_{\mathbb{R}^N} b(x)G(u)$$

on a C^1-submanifold M of $\mathcal{D}^{1,2}(\mathbb{R}^N)$ and prove that any local minimizer of the constrained functional $J|_M$ with respect to the V-topology must be a local minimizer with respect to the "bigger" $\mathcal{D}^{1,2}(\mathbb{R}^N)$-topology.

Second, we obtain constant-signed solutions as local minimizers in the V-topology on Nehari manifolds corresponding to suitably truncated nonlinearities, which we prove to be local minimizers in the topology of $\mathcal{D}^{1,2}(\mathbb{R}^N)$, and finally we get a third solution of Mountain-Pass type on the Nehari manifold corresponding to the original nonlinearity.

We note that when dealing with problems in $\mathbb{R}^N$ one is faced with difficulties such as lack of compactness, among other things. Also, unlike many papers in $\mathbb{R}^N$ that use spaces like $H^1(\mathbb{R}^N)$ having 'nicer' properties than $\mathcal{D}^{1,2}(\mathbb{R}^N)$, we cannot do that here due to the zero-mass situation, cf. the recent papers [3], [4], whose ideas motivated the present work. And our use of manifolds was inspired by the paper [5] which dealt with bounded domains. Finally, we note that our results extend to the whole space $\mathbb{R}^N$ the pioneering works of Brezis-Nirenberg [2], Ambrosetti-Rabinowitz [1] and Z.Q. Wang [6]. We list below some references.

Joint work with Sigfried Carl and Hossein Tehrani.

References

[1] A. Ambrosetti, P.H. Rabinowitz, *Dual variational methods in critical point theory and applications*, J. Funct. Anal. 14 (1973), 349–381.

[2] H. Brezis, L. Nirenberg, *H^1 versus C^1 minimizers*, C. R. Acad. Sci. Paris 317 (1993), 465–472.

[3] S. Carl, D.G. Costa, H. Tehrani, *Extremal and sign-changing solutions of supercritical logistic-type equations in $\mathbb{R}^N$*, Calc. Var. Partial Differential Equations 54 (2015), 4143–4164.

[4] S. Carl, D.G. Costa, H. Tehrani, *$\mathcal{D}^{1,2}(\mathbb{R}^N)$ versus $C(\mathbb{R}^N)$ local minimizer on manifolds and multiple solutions for zero-mass equations in $\mathbb{R}^N$*, DOI: 10.1515/acv-2016-0012 (2017), 4143–4164.

[5] H. Tehrani, *H^1 versus C^1 local minimizers on manifolds*, Nonlinear Anal. 26 (1996), 1491–1509.

[6] Z.-Q. Wang, *On a superlinear elliptic equation*, Ann. Inst. Henri Poincaré 8 (1991), 43–57.

Ground state solutions of Hamiltonean system in dimension 2

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Department of Mathematics, State University of Campinas, Brazil

Abstract.
Using the Pankov Generalised Nehari Manifold, we prove the existence of ground state solutions for Hamiltonean Systems in dimension 2, with non-linearities of the Trudinger-Moser type of critical growth. We consider the case of a bounded domain as well as the whole of $\mathbb{R}^2$.

Joint work with J. M. do O and J. Zhang

Uniqueness and symmetry results for nonlinear elliptic PDEs obtained with the help of linear and nonlinear flows

Maria J. Esteban (esteban@ceremade.dauphine.fr)
Paris Dauphine University

Abstract. In this talk I will present some cases where the use of (non)linear flows allows to identify the positive solutions of nonlinear elliptic PDEs, as well as the extremal functions for functional inequalities. The flow methods prove to be optimal and robust when allowed to deal with different equations and inequalities.

The difficulty in these problems is to find a flow along which the associated energy functional is non-increasing. The proofs of uniqueness does not necessarily use the all-time existence and the well-posedness of the flow, but the flow is used as a direction or guide along which to perturb the positive solutions of the equations, that is, the critical points of the corresponding energy functionals.

Self-similarity and time-weighted estimates for wave equations with singular potentials

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Abstract. We show time-weighted estimates in Lorentz spaces for the linear wave equation with singular potential. As a consequence, assuming radial symmetry on initial data and potentials, we obtain well-posedness of global solutions in critical weak-L^p spaces for semilinear wave equations. In particular, we can consider the Hardy potential $V(x) = c|x|^{-2}$ for small $|c|$. Self-similar solutions are obtained for potentials and initial data with the right homogeneity. Our approach relies on performing estimates in the predual of weak-L^p, i.e., the Lorentz space $L^{(p',1)}$.

Joint work with Marcelo F. de Almeida (Federal University of Sergipe, Brazil).

Existence of bounded variation solutions for a 1-Laplacian problem with vanishing potentials

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Abstract. In this work it is studied a quasilinear elliptic problem in the whole space $\mathbb{R}^N$ involving the $1-$Laplacian operator, with potentials which can vanish at infinity. The Euler-Lagrange functional is defined in a space whose definition resembles $BV(\mathbb{R}^N)$. It is proved the existence of a nonnegative nontrivial bounded variation solution and the proof relies on a version of the Mountain Pass Theorem without the Palais-Smale condition to Lipschitz continuous functionals.

References

[1] C. Alves and M.T.O. Pimenta, *On existence and concentration of solutions to a class of quasilinear problems involving the* $1-$*Laplace operator* , arXiv: 1702.06718.1977 - 1991 (2013).

[2] M. Degiovanni and P. Magrone, *Linking solutions for quasilinear equations at critical growth involving the* $1-$*Laplace operator*, Calc. Var., 36, 591 - 609 (2009).

[3] G.M. Figueiredo and M.T.O. Pimenta, *Nehari method for locally Lipschitz functionals with examples in problems in the space of bounded variation functions*, to appear.

[4] G.M. Figueiredo and M.T.O. Pimenta, *Strauss' and Lions' type results in* $BV(\mathbb{R}^N)$ *with an application to* $1-$*Laplacian problem* , to appear.

[5] B. Kawohl and F. Schuricht , *Dirichlet problems for the* $1-$*Laplace operator, including the eigenvalue problem*, Commun. Contemp. Math., 9, No. 4, 525 - 543 (2007).

A minimaxmax problem for improving the torsional stability of rectangular plates

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Abstract. We use a gap function in order to compare the torsional performances of different reinforced plates under the action of external forces. Then we address a shape optimization problem whose target is to minimize the torsional displacements of the plate: this leads us to set up a minimaxmax problem, which includes a new kind of worst-case optimization. The reinforcement aims to strengthen the plate. The corresponding Euler-Lagrange equation cannot be written in strong form. We study the existence of optima within suitable classes of external forces and reinforcements. Our results are complemented with numerical experiments and with a number of open problems and conjectures.

Joint work with P. Antunes, D. Buoso, E. Berchio, D. Zucco.

Elliptic problems with a gradient term

Jean-Pierre Gossez (gossez@ulb.ac.be)
Université Libre de Bruxelles (ULB), Belgium

Abstract. We investigate the problem

$$(P) \qquad \begin{cases} -\Delta_p u = g(u)|\nabla u|^p + f(x,u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

in a bounded smooth domain $\Omega \subset \mathbb{R}^N$. Using a Kazdan-Kramer change of variable, we reduce this problem to a quasilinear one without gradient term and therefore approachable by variational methods. Among other results, we investigate the validity of the Ambrosetti-Rabinowitz condition. Existence and multiplicity results for (P) are established in several situations.

Joint work with D. de Figueiredo, H. Ramos Quoirin and P. Ubilla.

On the time-global bounds of Sobolev norms of global solutions of parabolic equation involving critical Sobolev exponent

Michinori Ishwiata (ishiwata@sigmath.es.osaka-u.ac.jp)
Osaka University

Abstract. In this talk, the existence of time-global bounds of the Sobolev norm of time global solutions for the following semilinear parabolic equation involving critical Sobolev exponent will be discussed:

$$(\mathrm{P}) \quad \begin{cases} \partial_t u &= \Delta u + u|u|^{p-2} & \text{in} \quad \mathbb{R}^N \times (0, T_m), \\ u|_{t=0} &= u_0 & \text{in} \quad \mathbb{R}^N, \end{cases}$$

where $N \geq 3$, $u_0 \in H^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ (for the simplicity) and T_m denotes the maximal existence time of classical solution of (P) and we only consider the case $T_m = \infty$. It is well-known that in the subcritical case (the case $p < 2^* := \frac{2N}{N-2}$), every (time-global) solution has a time-global bounds:

$$(\mathrm{GB}) \qquad \sup_{t>0} \|\nabla u(t)\|_2 < \infty.$$

This is based on the fact "$\|\nabla u(t)\|_2$ cannot oscillate too much" due to the subcriticality. On the other hand, in the critical case ($p = 2^*$), the above argument does not work and the time-global boundedness remains an open problem.

In this talk, we will prove (GB) actually holds for the critical case $p = 2^*$. The method of proof relies on the profile decomposition and the "extended" LaSalle principle to the noncompact situation.

Existence, nonexistence and multiplicity of positive solutions for the poly-Laplacian and nonlinearities with zeros

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Abstract. In this speech we consider the problems

$$\begin{cases} (-\Delta)^k u = \lambda f(x,u) + \mu g(x,u) & \text{in} \quad \Omega, \\ u > 0 & \text{in} \quad \Omega, \\ (-\Delta)^i u = 0 & \text{on} \quad \partial\Omega, \ i = 0, .., k-1 \end{cases} \tag{5}$$

where and $k \in \mathbb{N}$, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary, $\lambda, \mu \geq 0$ are two parameters and f, g are continuous nonnegative functions.

The main interest is when the nonlinearity is nonnegative but admits a zero and f, g are, respectively, identically zero above and below the zero. We first prove the existence of multiple positive solutions when the parameters lie in a region of the form $\lambda > \overline{\lambda}$ and $0 < \mu < \overline{\mu}(\lambda)$.

Based on the observation that this result is sometimes in contrast with the previously studied Laplacian or p-Laplacian cases, we then show that, in some cases, existence and multiplicity can be extended to hold without the bound on μ, while in other cases, existence is actually lost for large values of μ.

On a class of supercritical elliptic problem in hyberbolic space

Olímpio Miyagaki (ohmiyagaki@gmail.com)
Universidade Federal de Juiz de Fora

Abstract. In ths talk, I would like to discuss a class of supercritical problems in hyberbolic space, as example, the Hénon-type problem in hyperbolic space. The problem involves a logarithm weight in the Poincaré ball model, bringing singularities on the boundary. Considering radial functions, a compact Sobolev embedding result is proved, which extends a former Ni result, in [3], made for a unit ball in $\mathbb{R}^N$. Combining this compactness embedding with the Mountain Pass Theorem is established a result of the existence of positive solution, extending He result in [2]. The subjects of the talk are contained in the article [1].

Joint work with Paulo César Carrião (UNIOESTE) and Luiz Fernando O. Faria (UFJF)

References

[1] P. C. Carrião, L. F. O. Faria and O. H. Miyagaki, Semilinear elliptic equations of the Hnon-type in hyperbolic space. Communications in Contemporary Mathematics, v. 18, p. 1550026, 2015.

[2] H. He, The existence of solutions for Henon equation in hyperbolic space, Proc. Japan Acad. 89 (2013) 24–28.

[3] W.-M. Ni, A nonlinear Dirichlet problem on the unit ball and its applications, Indiana Univ. Math. J. 31(6) (1982) 801–807.

Blow up solutions for Yamabe type problems on Riemannian manifolds with boundary

Anna Maria Micheletti (a.micheletti@dma.unipi.it)
Università di Pisa, Italy

Abstract. We show some noncompactness results for the set of positive solutions for various Yamabe type equations on Riemannian manifold with boundary.

We are interested in building solutions which blow up at one point on the boundary.

A new variational principle, convexity and supercritical variational problems

Abbas Moameni (momeni@math.carleton.ca)
Carleton University, Ottawa, Canada

Abstract. A variational principle is introduced to provide a new formulation and resolution for several boundary value problems with a variational structure. This principle allows one to deal with problems well beyond the weakly compact structure. As a result, we study several super-critical local and non-local partial differential equations.

Harnack type inequality revisited: application to a free boundary problem in Orlicz-Sobolev spaces

Sergio H. Monari Soares (monari@icmc.usp.br)
Universidade de São Paul, Brazil

Abstract. In this work, we intend to revisit the Harnack inequality and as an application to show the existence and non degeneracy property of the solutions to a free boundary problem.

Joint work with Jefferson Abrantes Santos (UFCG)

Positive solutions of Lane Emden problems in dimension 2 and uniqueness in convex domains

Filomena Pacella (pacella@mat.uniroma1.it)
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Abstract. We present a few recent results about positive solutions of the classical Lane Emden equation in bounded domains of the plane, with Dirichlet boundary conditions and large exponent in the nonlinearity. A first one concerns the asymptotic profile and a precise quantization formula for the energy , as the exponent $p \to \infty$, of any family of positive solutions u_p satisfying : $p \int_\Omega |\nabla u_p|^2 \quad \to b$ for some $b \in \mathbb{R}$. Then we show that positive solutions concentrating only at one point have Morse index 1 . From this, using a result of C.S. Lin (1994), we deduce that, for large exponents, in planar convex domains there is only one positive solution satisfying the above energy bound, namely the least energy solution. In particular there are no multi-peak solutions.

Joint work with F.De Marchis, M.Grossi and I.Ianni.

Concentration along geodesics for a nonlinear Steklov problem arising in corrosion modelling

Carlo Pagani (carlo.pagani@polimi.it)
Politecnico di Milano

Abstract.
We consider the problem of finding pairs $(\lambda, \mathfrak{u})$, with $\lambda > 0$ and $\mathfrak{u}$ a harmonic function in a three dimensional torus-like domain $\mathcal{D}$, satisfying the nonlinear boundary condition $\partial_\nu \mathfrak{u} = \lambda \sinh \mathfrak{u}$ on $\partial\mathcal{D}$. This type of boundary condition arises in corrosion modelling (Butler Volmer condition). We prove existence of solutions which concentrate along some geodesics of the boundary $\partial\mathcal{D}$ as the parameter λ goes to zero.

Joint work with Dario Pierotti, Angela Pistoia and Giusi Vaira.

A nonhomogeneous semilinear elliptic equation

Francisco Odair de Paiva (odair@dm.ufscar.br)
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Abstract.

For a bounded domain Ω, $p > 1$ and a nonnegative L^1-function h in Ω which is strictly positive in a open subset, we prove that there exists a solution of the semilinear elliptic problem

$$\begin{cases} -\Delta u & = & \lambda u - h|u|^{p-1}u + f, & \text{in } \Omega \\ u & = & 0, & \text{on } \partial\Omega, \end{cases}$$

for every $\lambda \in \mathbb{R}$ and $f \in L^2(\Omega)$.

Joint work with David Arcoya (Granada) and José M. Mendoza (São Carlos)

Solitary waves for a nonlocal model

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Abstract. We study existence and behavior of solitary waves for a nonlocal model describing two-dimensional weakly nonlinear long-wave perturbations on the background of a boundary-layer type plane-parallel shear flow without inflection points. The existence result is accomplished with an application of the mountain pass theorem. Regularity and decay properties are then established taking into account the convolution form of the equation and bootstrap type arguments.

Oscillating Solutions for nonlinear Helmholtz Equations

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Abstract. The aim of this talk is to present some existence results of radially symmetric oscillating solutions for a class of nonlinear autonomous Helmholtz equations and in particular to analyse their exact asymptotic behavior at infinity. Moreover, some generalizations to non-autonomous radial equations as well as existence results for non-radial solutions will be discussed. These results are linked with the existence of standing waves solutions of nonlinear wave equations with large frequencies.

Joint work with Rainer Mandel (Karlsruher Institut für Technologie) and Eugenio Montefusco ("Sapienza" Università di Roma)

Regularity theory for the Isaacs equations: an approximation method

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Abstract. In this talk, we consider an Isaacs equation and study the regularity of solutions in Sobolev and Hölder spaces. This class of equations arises in the study of two-players, zero-sum, stochastic differential games. In addition, it is a toy-model for non-convex/non-concave operators. In the framework of viscosity solutions, fundamental developments regarding the Isaacs equation have been produced; for example, the existence and uniqueness of solutions. We propose an approximation method, relating the Isaacs operator with a Bellman one. From a heuristic viewpoint, we import regularity from the latter to our problem of interest, by imposing a *proximity regime*. Distinct regimes yield different classes of estimates, covering the cases of Sobolev and Hölder spaces. We close the talk with some consequences and applications of our results.

Concave-convex problems refuse to die

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Abstract. We study the following elliptic problem $-A(u) = \lambda u^q$ with Dirichlet boundary conditions, where $A(u)(x) = \Delta u(x)\chi_{D_1}(x) + \Delta_p u(x)\chi_{D_2}(x)$ is the Laplacian in one part of the domain, D_1, and the $p-$Laplacian (with $p > 2$) in the rest of the domain, D_2. We show that this problem exhibits a concave-convex nature for $1 < q < p-1$. In fact, we prove that there exists a positive value λ^* such that the problem has no positive solution for $\lambda > \lambda^*$ and a minimal positive solution for $0 < \lambda < \lambda^*$. If in addition we assume that p is subcritical, that is, $p < 2N/(N-2)$ then there are at least two positive solutions for almost every $0 < \lambda < \lambda^*$, the first one (that exists for all $0 < \lambda < \lambda^*$) is obtained minimizing a suitable functional and the second one (that is proven to exist for almost every $0 < \lambda < \lambda^*$) comes from an appropriate mountain pass argument.

Joint work with A. Molino (U. Granada, Spain).

References

[1] A. Ambrosetti, J. García-Azorero, I. Peral, Multiplicity results for some nonlinear elliptic equations, *J. Funct. Anal.* 137 (1996), 219–242.

[2] F. Charro, E. Colorado, I. Peral, Multiplicity of solutions to uniformly elliptic Fully Nonlinear equations with concave-convex right hand side, *J. Differential Equations.* 246 (2009), 4221–4248.

[3] J. García–Azorero, I. Peral, J. D. Rossi, A convex-concave problem with a nonlinear boundary condition, *J. Differential Equations.* 198 (1) (2004), 91–128.

[4] J. García–Azorero, J.J. Manfredi and I. Peral, Sobolev versus Hlder local minimizers and global multiplicity for some quasilinear elliptic equations, *Comm. Contemp. Math.*, 2(3) (2000), 385–404.

On the Trudinger-Moser inequality for fractional Sobolev spaces

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Abstract.
We prove the validity of the Trudinger-Moser inequality

$$\sup\Big\{\int_\Omega e^{|u|^{N/(N-s)}}dx \ ;\ u\in W_0^{s,p}(\Omega)\ ,\ [u]_{W^{s,p}(\mathbb{R}^N)}\le 1\Big\}<\infty$$

when $\alpha\in[0,\alpha_0)$ for some $\alpha_0>0$. Here Ω is a bounded domain of $\mathbb{R}^N(N\ge 2)$, $s\in(0,1), sp=N$, $W_0^{s,p}(\Omega)$ is a Sobolev-Slobodeckij space, and $[\,.\,]_{W^{s,p}(\mathbb{R}^N)}$ is the associated Gagliardo seminorm. Moreover, we exhibit an explicit exponent $\alpha^*_{s,N}>0$, which does not depend on Ω, such that the Trudinger-Moser inequality does not hold true for $\alpha\in(\alpha^*_{s,N},+\infty)$.

Joint work with Enea Parini (University of Aix-Marseille)

Adams' inequality (with the exact growth condition)

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Abstract. Adams' inequality is the complete generalization of the Trudinger-Moser inequality to the case of Sobolev spaces involving higher order derivatives. In this talk we discuss the optimal growth rate of the exponential-type function in Adams' inequality when the problem is considered in the whole space $\mathbb{R}^n$.

Joint work with Nader Masmoudi.

Some results of existence of solutions to a quasilinear defocusing Schrödinger equation

Carlos Alberto Santos (csantos@unb.br)
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Abstract. In this talk, we are going to present some results of existence of solutions to the quasilinear defocusing Schrödinger equation

$$-\Delta u + \frac{k}{2} u \Delta u^2 + V(x)u = g(x,u) \text{ in } \mathbb{R}^N, \tag{6}$$

where $k \in \mathbb{R}$ is a real parameter, and $V(x)$ is a non-negative continuous potential. Let us present conditions under $V(x)$ and $g(x,u)$ to show existence of bounded nodal solutions ($u^+, u^- \neq 0$) and explosive solutions ($u(x) \to \infty$ as $|x| \to \infty$) to equation (6).

Joint work with Jiazheng Zhou (University of Brasília, Brazil) and Minbo Yang (Zhejiang Normal University, People's Republic of China).

A Landesman-Lazer local condition for semilinear elliptic problems

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Abstract. In this talk we present results on existence, multiplicity and nonexistence of solutions for semilinear elliptic problems dependending on a parameter under a Landesman-Lazer local condition. In our hypotheses there is no growth restriction at infinity on the nonlinear term. Furthermore, this term may change sign. In order to establish the existence of solution, we first truncate the nonlinear term, then we combine the Lyapunov-Schmidt Reduction Method with an approximation argument based on the bootstrap technique. Related results for semilinear elliptic problems with the nonlinear term depending on the gradient are also presented.

Optimal boundary Harnack estimates and a priori bounds for elliptic inequalities

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Abstract. We prove optimal boundary versions of some basic estimates from the regularity theory of uniformly elliptic PDE, such as growth lemmas and half-Harnack inequalities, and show how such estimates can be used to obtain new and optimal a priori bounds for positive sub- and super-solutions of a class of elliptic equations, both in divergence and in non-divergence form, involving a superlinear nonlinearity.

References

[1] Boundary weak Harnack estimates and quantitative strong maximum principles for uniformly elliptic PDE , to appear in International Mathematics Research Notices, arXiv:1608.01359 ,

Some weakly coercive quasilinear problems with forcing

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Abstract. We consider the forced problem

$$-\Delta_p u - V(x)|u|^{p-2}u = f(x),$$

where $\Delta_p u$ is the p-Laplacian ($1 < p < \infty$) in a domain $\Omega \subset \mathbb{R}^N$, $V \geq 0$ and

$$Q_V(u) := \int_\Omega |\nabla u|^p \, dx - \int_\Omega V|u|^p \, dx > 0 \quad \text{for all test functions } u \in \mathcal{D}(\Omega) \setminus \{0\}.$$

We show that under some additional conditions on Q_V this problem has a solution for all f in a suitable space of distributions. Then we apply this result to some classes of functions V which in particular include the Hardy potential $V(x) = \left(\frac{N-p}{p}\right)^p |x|^{-p}$ and the potential $V(x) = \lambda_{1,p}(\Omega)$, where $\lambda_{1,p}(\Omega)$ is the Poincaré constant on the strip $\Omega = \omega \times \mathbb{R}^M$, $\omega \subset \mathbb{R}^{N-M}$ bounded.

Joint work with Michel Willem.

Critical and subcritical fractional Trudinger-Moser type inequalities on $\mathbb{R}$

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Abstract. In this talk, we concern with the critical and subcritical Trudinger-Moser type inequalities for functions in a fractional Sobolev space $H^{1/2,2}$ on the whole real line. More precisely, put

$$A(\alpha) = \sup_{\substack{u \in H^{1/2,2}(\mathbb{R}) \setminus \{0\} \\ \|(-\Delta)^{1/4} u\|_{L^2(\mathbb{R})} \le 1}} \frac{1}{\|u\|_{L^2(\mathbb{R})}^2} \int_{\mathbb{R}} \left(e^{\alpha u^2} - 1 \right) dx.$$

Then we prove

$$A(\alpha) < +\infty, \text{ if } \alpha < \pi, \qquad A(\alpha) = +\infty, \text{ if } \alpha \ge \pi$$

and $A(\alpha)$ is attained for any $\alpha \in (0, \pi)$. Also we show that a relation

$$B(\pi) = \sup_{\alpha \in (0,\pi)} \frac{1 - (\alpha/\pi)}{(\alpha/\pi)} A(\alpha),$$

where

$$B(\pi) = \sup_{\substack{u \in H^{1/2,2}(\mathbb{R}) \\ \|u\|_{H^{1/2,2}(\mathbb{R})} \le 1}} \int_{\mathbb{R}} \left(e^{\pi u^2} - 1 \right) dx < \infty.$$

This talk is based on a preprint [1].

References

[1] F. Takahashi: *Critical and subcritical fractional Trudinger-Moser type inequalities on* $\mathbb{R}$, arXiv:1702.08206.

Sharp Sobolev-Lorentz embeddings and Hardy inequality

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Abstract. In 1977 Alvino proved the sharp inequality

$$\|u\|_{p^*,q} \leq \frac{p}{n-p}\,\omega_n^{-\frac{1}{n}}\,\|\nabla u\|_{p,q}, \quad 1 \leq p < n, 1 \leq q \leq p \tag{7}$$

related to the optimal embedding for the Sobolev-Lorentz spaces $\mathcal{D}^1 L^{p,q}(\mathbb{R}^n)$ into the Lorentz spaces $L^{p^*,q}(\mathbb{R}^n)$. In the particular case $q = p$, corresponding to the optimal Sobolev embedding of $\mathcal{D}^{1,p}(\mathbb{R}^n)$ into $L^{p^*,p}(\mathbb{R}^n)$, inequality (7) turns out to be equivalent to the classical Hardy inequality

$$\left(\frac{n-p}{p}\right)^p \int_{\mathbb{R}^n} \frac{|u|^p}{|x|^p} dx \leq \int_{\mathbb{R}^n} |\nabla u|^p dx$$

whose sharp constant is not achieved.

In this talk we will extend the validity of (7) to any value $p \leq q \leq \infty$, observing that the sharp constant, which does not depend on q, is never attained if $q < \infty$: we will also prove that all the scale of Sobolev Lorentz embeddings is a consequence of Hardy inequality. Surprisingly, in the case $q = \infty$ we will obtain the reverse implication, proving that inequality (7) is actually equivalent to Hardy inequality: the new equivalent version of Hardy inequality

$$\|v\|_{p^*,\infty} \leq \frac{p}{n-p}\,\omega_n^{-\frac{1}{n}}\,\|\nabla v\|_{p,\infty}$$

is achieved, and a maximizer is given by $v(x) = |x|^{-\frac{n-p}{p}}$.

Joint work with D. Cassani (Università degli Studi dell'Insubria) and B. Ruf (Università degli Studi di Milano).

A nonlinear heat equation with exponential nonlinearity in $\mathbb{R}^2$

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Abstract. We consider a semilinear heat equation with exponential nonlinearity in $\mathbb{R}^2$. We prove that local solutions do not exist for certain data in an Orlicz space related to the nonlinearity, even though a small data global existence result holds in the same space. Then we focus on a specific exponential nonlinearity and we show that a particular singular initial datum yields the threshold between existence and non-existence of solutions.

Joint work with N. Ioku (Ehine University, Japan) and B. Ruf (Università di Milano, Italy).

On a class of Kirchhoff elliptic equations involving critical growth and vanishing potentials

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Abstract. We establish the existence of positive solutions for a class of stationary Kirchhoff type equations defined in the whole $\mathbb{R}^3$ involving critical growth in the sense of the Sobolev embedding and potentials which decay to zero at infinity in some direction. In order to obtain the solution we used minimax techniques combined with an appropriated truncated argument, and a priori estimates. This results are new even for the local case which corresponds to nonlinear Schrödinger equations.

Joint work with J.M. do Ó (UFPB) and M.A. Souto (UFCG).

Long time behavior of the solution of the Dirichlet problem for the porous medium equation in exterior domains

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Abstract. I will present recent results on the long time behavior of the solution to the Cauchy-Dirichlet problem in an exterior 1 or 2 dimensional domain with integrable and compactly supported initial datum.

I will begin by stating the corresponding result by Brandle, Quiros and Vazquez in the case of exterior $N-$dimensional domains, $N \geq 3$ (cf. [1]).

In small dimensions the long time behavior is more involved since the rate of decay of the solutions in compact sets differs from the global one.

Global results are known in 1 dimension (cf. [3]) but they don't give the right decay rate nor the final profile in compact sets.

On the other hand, in dimension 2, the long time behavior is known only in "exterior" domains of the form $\{|x| \geq \xi t^{1/2m}(\log t)^{(m-1)/2m}\}$ (cf. [2]).

I will present recent work in collaboration with Carmen Cortazar and Fernando Quiros where we found both the decay rate (which depends on the spatial scale) and the final profile in dimensions 1 and 2 (cf. [4, 5]).

References

[1] Brändle, C.; Quirós, F.; Vázquez, J. L. *Asymptotic behaviour of the porous media equation in domains with holes*. Interfaces Free Bound. 9 (2007), no. 2, 211–232.

[2] Gilding, B. H.; Goncerzewicz, J. *Large-time behaviour of solutions of the exterior-domain Cauchy-Dirichlet problem for the porous media equation with homogeneous boundary data*. Monatsh. Math. 150 (2007), no. 1, 11–39.

[3] Kamin, S.; Vázquez, J. L. *Asymptotic behaviour of solutions of the porous medium equation with changing sign*. SIAM J. Math. Anal. 22 (1991), no. 1, 34–45.

[4] Cortázar, C.; Quirós, F.; Wolanski, N. *Near field asymptotic behavior for the porous medium equation on the half-line*. To appear in Advanced Nonlinear Studies, arXiv:1603.05936 [math.AP].

[5] Cortázar, C.; Quirós, F.; Wolanski, N. *N*ear field asymptotics for the porous medium equation in exterior domains. The critical two-dimensional case. Preprint.

Abstracts of the posters

Existence of solutions for a fractional nonhomogeneous semilinear elliptic equation

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Abstract. For a bounded smooth domain Ω with Lipschitz boundary in $\mathbb{R}^n$, $n > 2s$, we consider the following problem:

$$\begin{cases} (-\Delta)^s u & = & \lambda u + g(x,u) - h|u|^{p-1}u + f, & \text{in } \Omega \\ u & = & 0, & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where for $s \in (0,1)$, $(-\Delta)^s$ is the nonlocal fractional Laplace operator and we assume that g is a bounded Carathéodory function in $\Omega \times \mathbb{R}$, $p > 1$ and h is a nonnegative measurable function in Ω which is strictly positive in a set of positive measure. And we show the existence of a solution of the problem considered for every $\lambda \in \mathbb{R}$ and $f \in L^2(\Omega)$, contrarly with the case $h \equiv 0$ for the classical laplacian operator, and extending the result of Arcoya, Paiva and Mendoza in [4].

References

[1] S. Alama and G. Tarantello, *On semilinear elliptic equations with indefinite nonlinearities.* Calc. Var. Partial Differential Equations 1 (1993), 439-475.

[2] S. Alama and G. Tarantello, *On the solvability of a semilinear elliptic equation via an associated eigenvalue problem.* Math. Z. 221 (1996), 467-493.

[3] S. Alama and G. Tarantello, *Elliptic problems with nonlinearities indefinite in sign.* J. Funct. Anal. 141 (1996), 159-215.

[4] D. Arcoya, F. O. de Paiva and J. M. Mendoza, *Existence of solutions for a nonhomogeneous semilinear elliptic equation*, preprint (2017).

[5] E. Di Nezza, G. Palatucci and E. Valdinoci, *Hitchhiker's guide to the fractional Sobolev spaces*, Bull. Sci. math. 136 (2012) 521-573.

[6] R. Servadei and E. Valdinoci, *Lewy-Stampacchia type estimates for variational inequalities driven by (non)local operators*, Rev. Mat. Iberoam. 29 (2013), no. 3, 1091-1126.

[7] R. Servadei and E. Valdinoci, *Variational methods for non-local operators of elliptic type*, 2010 AMS.

[8] M. Moustapha Fall and V. Felli, *Unique continuation property and local asymptotics of solutions to fractional elliptic equations*, Comm. Partial Differential Equations, 39(2):354-397, 2014.

[9] P.H. Rabinowitz, *Some Minimax Theorems and Applications to Nonlinear Partial Differential Equations*, in Nonlinear Analysis: A Collection of Papers in Honor of Erich Röthe, Academic Press, New York 1978, pp. 161-177.

Hénon type equations with jumping nonlinearities involving critical growth

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Abstract. In this work, we search for two non-trivial radially symmetric solutions of the Dirichlet problem involving a Hénon-type equation of the form

$$\begin{cases} -\Delta u &= \lambda u + |x|^\alpha k(u_+) + f(x) & \text{in} \quad B_1, \\ u &= 0 & \text{on} \quad \partial B_1, \end{cases} \tag{8}$$

where $\lambda > 0$, $\alpha \geq 0$, B_1 is a unity ball centered at the origin of $\mathbf{R}^N$ $(N \geq 3)$ and $k(s) = s^{2^*_\alpha - 1} + g(s)$ with $2^*_\alpha = 2(N+\alpha)/(N-2)$ and $g(s)$ is a C^1 function in $[0, +\infty)$ which is assumed to be in the subcritical growth range.

The proofs are based on variational methods and to ensure that the considered minimax levels lie in a suitable range, special classes of approximating functions which have disjoint support with Talenti functions (Hénon version) are constructed.

Joint work with João Marcos do Ó (UFPB) and Bruno Ribeiro (UFPB).

References

[1] A. Ambrosetti, G. Prodi, *On the inversion of some differentiable mappings with singularities between Banach spaces*, Ann. Mat. Pura Appl. **93**(1972) 231-246.

[2] H. Brezis, L. Nirenberg *Remarks on finding critical points*, Comm. Pure Appl. Math. **44**(1991) 939-963.

[3] M. Calanchi, B. Ruf, *Elliptic equations with one-sided critical growth*, Electron. J. Diff. Eq. **2002**(2002) 1-21.

[4] D. Figueiredo, J. Yang, *Critical superlinear Ambrosetti-Prodi problems*, Topol. Methods Nonlinear Anal. **14** (1999) 59-80.

[5] F. Gazzola, B. Ruf, *Lower-order perturbations of critical growth nonlinearities in semilinear elliptic equations*, Adv. Diff. Eq. **2**(1997) 555-572.

[6] M. Hénon, *Numerical experiments on the stability of spherical stellar systems*, Astronomy and Astrophysics **24** (1973) 229-238.

[7] J. Kazdan, F. Warner, *Remarks on some quasilinear elliptic equations*, Comm. Pure Appl. Math. **28** (1975) 567-597.

Isolated singularities of solutions for a critical elliptic system

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Abstract. In this paper we study the asymptotic behavior of local solutions for strongly coupled critical elliptic systems near an isolated singularity, when the dimension is less than or equal to five and the potential of the operator is less, in the sense of bilinear forms, than the geometric threshold potential of the conformal Laplacian. We prove a sharp result on the removability of the isolated singularity for all components of the solutions.

Joint work with João Marcos do Ó (UFPB) and Almir Santos (UFS)

References

[1] DRUET, O., HEBEY, E. AND VÉTOIS, J. - Bounded stability for strongly coupled critical elliptic systems below the geometric threshold of the conformal Laplacian., *Journal of Functional Analysis*, **258**, 999-1059, 2010.

[2] CHEN, Z AND LIN, C-S - Removable singularity of positive solutions for a critical elliptic ssystem with isolated singularity. *Mathematische Annalen*, **363**, 501-523, 2015.

[3] MARQUES, F.C. - Isolated singularities of solutions to the Yamabe equation. *Calculus of Variations*, **32**, 349-371, 2008.

Boundary value problems for a class of planar complex vector fields

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Abstract. Let L be a vector field defined in an open subset $\tilde{\Omega}$ of the plane satisfying the following property: For each point p, either L is elliptic at p or there exist local coordinates (x, y) centered at p such that L becomes (near p) a nonzero multiple of the vector field $\dfrac{\partial}{\partial y} - i|y|^{\sigma}\dfrac{\partial}{\partial x}$ for some $\sigma > 0$. In this study the focus is in the understanding of the boundary value problem $Lu = au + b\overline{u} + f$ in Ω, $\Re(\overline{\Lambda}u) = \varphi$ on $\partial\Omega$, on a simply connected domain $\Omega \subset\subset \tilde{\Omega}$ with $a, b, f \in L^p(\Omega)$ and $\Lambda, \varphi \in C^{\alpha}(\partial\Omega)$, $|\Lambda| = 1$, $0 < \alpha < 1$. The use of properties of an associate integral operator together with the Fredholm alternative, allows us to establish solvability when the index is nonnegative ([1]).

References

[1] C. Campana and A. Meziani. *Boundary value problems for a class of planar complex vector fields*, Jornal of Differential Equations, (2016), Volume 261, 10, 5609-5636.

A type of Brézis-Oswald problem to $\Phi-$*Laplacian* operator in the presence of singular terms

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Abstract. We are concerned in showing an existence result of solutions and a comparison principle for sub and super solutions in $W^{1,\Phi}_{loc}(\Omega)$ to the problem

$$\begin{cases} -\Delta_\Phi u = f(x,u) \text{ in } \Omega, \\ u > 0 \text{ in } \Omega, \ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where f has Φ-sublinear growth and may be singular at $u = 0$. Our results are an improvement and complement of the classical Brézis-Oswald and Diaz-Saa's results to Orlicz-Sobolev setting for singular nonlinearities. Some of our results are news even for the Laplacian operator setting.

Joint work with Goncalves, J. V., Santos, C. A. and Silva, E. D.

References

[1] H. Brézis, & L. Oswald, *Remarks on sublinear elliptic equations*, Nonlinear Anal., 10, 55-64, (1986)

[2] J. L. Díaz, J. E. Saa, *Existence et unicité de solutions positives pour certaines équations elliptiques quasilinéaires*, C.R.A.S. de Paris t. 305, Serie I , 521–524, (1987)

[3] J. V. Goncalves, M. L. Carvalho & C. A. Santos *Quasilinear elliptic systems with convex-concave singular terms and $\Phi-$ Laplacian operator*, To appear in Differential and Integral Equations.

Multiplicity of solutions to fourth-order superlinear elliptic problems under Navier conditions

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Abstract. We establish the existence and multiplicity of solutions for a class of fourth-order superlinear elliptic problems under Navier conditions on the boundary. Here we do not use the Ambrosetti-Rabinowitz condition; instead we assume that the nonlinear term is a nonlinear function which is nonquadratic at infinity.

References

[1] Furtado, M.F ; da Silva, Edcarlos, Superlinear elliptic problems under the Nonquadriticty condition at infinity *Proceedings of the Royal Society of Edinburgh: Section A Mathematics.*, (2015).

[2] Yang Pu, Xing-Ping Wu and Chun-Lei Tang, Fourth-order Navier boundary value problem with combined nonlinearities. *J. Math. Annal. 398: 798-813*, (2013).

[3] Chun Li, Zeng-Qi Ou, and Chun-Lei Tang, Existence and Multiplicity of Nontrivial Solutions for a Class of Fourth-Order Elliptic Equations. *Hindawi Publishing Corporation Abstract and Applied Analysis 101155: 597193*, (2013).

[4] Anna Maria Micheletti, Angela Pistoia Nontrivial solutions for some fourth order semilinear elliptic problems *Nonlinear Analysis, 34: 509–523*, (1998).

On Lane-Emden systems with singular nonlinearities and applications to MEMS

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Abstract. We analyse the Lane-Emden system

$$\begin{cases} -\Delta u = \frac{\lambda f(x)}{(1-v)^2} & \text{in } \Omega \\ -\Delta v = \frac{\mu g(x)}{(1-u)^2} & \text{in } \Omega \\ 0 \leq u, \quad v < 1 & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases} \tag{9}$$

where λ and μ are positive parameters and Ω is a smooth bounded domain of $\mathbb{R}^N$ $(N \geq 1)$. Here we prove the existence of a critical curve Γ which splits the positive quadrant of the (λ, μ)-plane into two disjoint sets $\mathcal{O}_1$ and $\mathcal{O}_2$ such that the problem (9) has a smooth minimal stable solution (u_λ, v_μ) in $\mathcal{O}_1$, while for $(\lambda, \mu) \in \mathcal{O}_2$ there are no solutions of any kind. We also establish upper and lower estimates for the critical curve Γ and regularity results on this curve if $N \leq 7$. Our proof is based on a delicate combination involving maximum principle and L^p estimates for semi-stable solutions of (9).

Joint work with João Marcos B. do Ó (UFPB)

References

[1] D. Cassani, J. M. do Ó, N. Ghoussoub, On a fourth order elliptic problem with a singular nonlinearity, Adv. Nonlinear Stud. **9** (2009), 189–209.

[2] C. Cowan, Regularity of the extremal solutions in a Gelfand system problem, Adv. Nonlinear Stud. **11** (2011), 695–700.

[3] C. Cowan, P. Esposito, N. Ghoussoub, A. Mordifam, The Critical Dimension for a Fourth Order Elliptic Problem with Singular Nonlinearity, Arch. Ration. Mech. Anal. **198** (2010), 763–787.

[4] C. Cowan, M. Fazly, Regularity of the extremal solutions associated to elliptic systems, J. Differential Equations **257** (2014), 4087–4107.

[5] C. Cowan, N. Ghoussoub, Regularity of semi-stable solutions to fourth order nonlinear eigenvalue problems on general domains, Calc. Var. Partial Differential Equations **49** (2014), 291–305.

[6] M. Crandall, P. Rabinowitz, Some continuation and variational methods for positive solutions of nonlinear elliptic eigenvalue problems, Arch. Ration. Mech. Anal. **58** (1975), 207–218.

[7] L. Dupaigne, A. Farina, B. Sirakov, Regularity of the extremal solutions for the Liouville system. Geometric partial differential equations, 139–144, CRM Series, 15, Ed. Norm., Pisa, 2013.

[8] P. Esposito, N. Ghoussoub, Y. Guo; Compactness along the branch of semistable and unstable solutions for an elliptic problem with a singular nonlinearity, Comm. Pure Appl. Math. **60** (2007), no. 12, 1731-1768.

[9] P. Esposito, N. Ghoussoub, Y. Guo, Mathematical analysis of partial differential equations modelling electrostatic MEMS, Courant Lecture Notes in Mathematics, 2010.

[10] M. Fazly, N. Ghoussoub, De Giorgi's type results for elliptic systems, Calc. Var. PDE **47** (2013), no. 3-4, 809–823.

[11] N. Ghoussoub, Y. Guo, On the partial differential equations of electrostatic MEMS device: stationary case, SIAM J. Math. Anal. **38** (2007), 1423–1449.

[12] M. Montenegro, Minimal solutions for a class of elliptic systems, Bull. London Math. Soc. **37** (2005), 405–416.

Existence of radial graphs with prescribed curvature and boundary

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Abstract. We consider the problem of finding hypersurfaces of constant curvature and prescribed boundary in the Euclidean space, without assuming the convexity of the prescribed solution and using the theory of fully nonlinear elliptic equations as the main tool to solve this problem. If the given data admits a suitable radial graph as a subsolution, then we prove that there exists a radial graph with constant curvature and realizing the prescribed boundary. As an application, it is proved that if $\Omega \subset \mathbb{S}^n$ is a mean convex domain whose closure is contained in an open hemisphere of $\mathbb{S}^n$ then, for $0 < R < n(n-1)$, there exists a radial graph of constant scalar curvature R and boundary $\partial\Omega$.

This work was presented in [1].

References

[1] Cruz, F., Radial graphs of constant curvature and prescribed boundary. *Calc. Var. Partial Differential Equations,* 56 (2017).

The IVP for the Benjamin-Ono-Zakharov-Kuznetsov equation in low regularity Sobolev spaces

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Abstract. We establish local well-posedness for the initial value problem (IVP) associated to the Benjamin-Ono-Zakharov-Kuznetsov equation

$$u_t + \mathcal{H}\partial_x^2 u + u_{xyy} + uu_x = 0, \;\; (x, y) \in \mathbb{R}^2, \; t > 0,$$

in Sobolev spaces $H^s(\mathbb{R}^2)$, where $s > 11/8$, see [1]. To this we take account the approach of Koch and Tzvetkov for the Benjamin-Ono equation, in [2].

Joint work with Ademir Pastor (IMECC-UNICAMP).

References

[1] A. Cunha and A. Pastor, The IVP for the Benjamin-Ono-Zakharov-Kuznetsov equation in low regularity Sobolev spaces, *Journal of Differential Equations (Print)*, v. 261, p. 2041–2067, 2016.

[2] Koch, H and Tzvetkov, N, On the local well-posedness of the Benjamin-Ono equation in $H^s(\mathbb{R})$. *Int. Math. Res. Not. IMRN.*, 1449–1464, 2003.

Existence of nontrivial solutions for a Kirchhoff-Schrödinger type equation in $\mathbb{R}^4$ with critical growth

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Abstract. We establish the existence of nontrivial weak solutions for a Kirchhoff-Schrodinger type problem in $\mathbb{R}^4$ involving a critical nonlinearity. The competition between the term due to the nonlocal coefficient and that one due to the nonlinearity turns out to be rather interesting. Our main tools are the variational method and the Concentration Compactness Principle.

Joint work with Francisco S. B. Albuquerque (UEPB).

References

[1] C.O. Alves & G.M. Figueiredo, Nonlinear perturbations of a periodic Kirchhoff equation in $\mathbb{R}^N$, Nonlinear Anal. 75 (2012), 2750-2759

[2] L. Jeanjean, On the existence of bounded Palais-Smale sequences and applications to a Landesman-Lazer type problem set on $\mathbb{R}^N$, Proc. Roy. Soc. Edinburgh Sect. A 129 (1999), 787-809.

[3] G. Kirchhoff, Mechanik, Teubner, Leipzig, 1883.

[4] J.L. Lions, On some questions in boundary value problems of mathematical physics, in: Contemporary Developments in Continuum Mechanics and Partial Differential Equations, Proc. Internat. Symppos., Inst. Mat. Univ. Fed. Rio de Janeiro, Rio de Janeiro, 1977, in: North-Holland Math. Stud., vol 30, North-Holland, Amsterdam, 1978, pp. 284-346.

[5] D. Naimen, The critical problem of Kirchhoff type elliptic equations in dimension four, J. Differential Equations 257 (2014), 1168-1193.

[6] J. Nie, Existence and multiplicity of nontrivial solutions for a class of Schrödinger-type equations, J. Math. Anal. Appl. 417 (2014), 65-79.

On a class of quasilinear Schrdinger equations with superlinear or asymptotically linear terms

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Abstract. We study the existence and nonexistence of nonzero solutions for the following class of quasilinear Schrödinger equations:

$$-\Delta u + V(x)u + \frac{\kappa}{2}[\Delta(u^2)]u = h(u), \quad x \in \mathbb{R}^N,$$

where $\kappa > 0$ is a parameter, $V(x)$ is a continuous potential which is large at infinity and the nonlinearity $h(t)$ can be asymptotically linear or superlinear at infinity. In order to prove our existence result we have applied minimax techniques together with careful $L^\infty-$estimates. Moreover, we prove a Pohozaev identity which justifies that $2^* = 2N/(N-2)$ is the critical exponent for this class of problems and it is also used to show nonexistence results.

Joint work with Edcarlos D. da Silva (UFG) and Uberlandio B. Severo (UFPB).

References

[1] U.B. Severo, E. Gloss and E.D. da Silva, *On a class of quasilinear Schrdinger equations with superlinear or asymptotically linear terms*, J. Differential Equations **263**, (2017), 3550-3580.

Instability of excited states of the nonlinear Schrödinger equation (NLS) with δ-interaction on a star graph

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Abstract. We investigate an orbital stability of the standing waves $e^{i\omega t}\mathbf{\Phi}(x)$ of the NLS equation with δ-interaction and two types of nonlinearities (power and logarithmic) on a star graph $\mathcal{G}$ (i.e. N half-lines joined at the vertex $\nu = 0$).

All possible profiles $\mathbf{\Phi}(x)$ generate the family of $[\frac{N-1}{2}] + 1$ solutions (stationary states) to the stationary equation associated with NLS-δ equation. There is a unique ground state among them (in the sense of constraint minimality of the associated action functional). It is the only solution symmetric modulo rotations of the edges of the graph $\mathcal{G}$. The rest of the stationary states are of different action and energy (or they are excited states).

Using the known theory by M. Grillakis, J. Shatah, W. Strauss, the theory of extensions of symmetric operators, and the spectral theory of self-adjoint Schrödinger operators, we study instability of excited states when an intensity of the δ-interaction is negative.

Critical quasilinear elliptic problems using concave-convex nonlinearities

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Abstract. In this work we deal with existence, multiplicity and asymptotic behaviour of nonnegative solutions of the problem

$$-\Delta_\Phi u = \lambda a(x)|u|^{q-2}u + b(x)|u|^{\ell^*-2}u \quad \text{in} \quad \Omega, \quad u = 0 \,\text{on}\, \partial\Omega. \tag{10}$$

where Δ_Φ denotes the $\Phi-$laplacian operator, which is defined by $\Delta_\Phi u = \mathrm{div}(\phi(|\nabla u|)\nabla u)$. One of these solutions is obtained as ground state solution by applying the well known Nehari method. The nonlinear term is a concave-convex function which presents a critical behavior at infinity. The concentration compactness principle is used in order to recover the compactness required in variational methods.

Joint work with Carvalho, M. L. M., Silva, E. D. and Gonçalves, J. V.

References

[1] A. Ambrosetti, H. Brézis, G. Cerami, *Combined effects of concave and convex nonlinearities in some elliptic problems*, J. Func. Anal., 122 (1994), 519-543.

[2] K. J. Brown, Y. Zhang, *The Nehari manifold for semilinear elliptic equation with a sign-changing weight function.* Jornal Differential Equation **193**, (2003), 481–499.

[3] G. Tarantello,*On nonhomogenous elliptic equations involving critical Sobolev expoent.* vol. 9. Ann. Inst. H. Poincare Anal. Non Lineaire **3**, (1992), 281-304.

Bifurcation of an Equation of Nonlocal Evolution restricted to space $L^2(T^N)$

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Abstract. Consider the equation of nonlocal evolution

$$\frac{\partial u(z,t)}{\partial t} = -u(z,t) + g(\lambda(J*u)(z,t))$$

restricted to space $L^2(T^N)$, where $(J*u)(z,t) = \frac{1}{(2\tau)^N}\int_{T^N} J(zw^{-1})u(w,t)dw$. We begin with λ a parameter larger than one, u a function defined in $R^n \times [0,+\infty)$, J a function of C^1 in R^n, even, non-negative, whose integral over $\Re^n$ is equal one and whose support is contained in $[-1,1]\times ... \times[-1,1]$ (n-times). Then the non-local evolution equation was considered up to space $P_{2\tau}$(limited and periodic continuous functions of term 2τ). In this work we prove the existence of local bifurcation with symmetry of the solutions of the above equation from the trivial equilibrium based on Equivariant Branching-Lemma and under certain conditions this local bifurcation is of the type Pitchfork.

Joint work with Antônio Luiz Pereira (IME/USP).

References

[1] A. Coutinho. *Local and global bifurcations of solutions of a non-local evolution equation*. 2008. 67f. Thesis (PhD in Mathematics)-Institute of Mathematics and Statistics, University of São Paulo, São Paulo, 2008.

[2] A. Vanderbauwhede. *Local bifurcation and symmetry*. Pitman Advanced Publishing Program, London, 1982.

[3] M. Golubitsky. *Singularities and Groups in Bifurcation Theory*, vol I and vol II. Springer-Verlag,Harper-Row, New York, 1985.

Concentration of positive solutions for a saturable coupled Schrödinger system

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Abstract. We consider the following saturable coupled Schrödinger system with two competing potential functions, for $N \geq 3$:

$$\begin{cases} -\varepsilon^2 \Delta u + a(x)u &= \dfrac{u^2+v^2}{1+s(u^2+v^2)}u + \lambda v \\ -\varepsilon^2 \Delta v + b(x)v &= \dfrac{u^2+v^2}{1+s(u^2+v^2)}v + \lambda u \end{cases}$$

with $u(x), v(x) \to 0$ when $|x| \to \infty$ and $u(x), v(x) > 0 \ \forall x \in \mathbb{R}^N$. Here, $0 < s < 1$ and $0 < \lambda < 1$ are given parameters. We prove the existence of bound states (solutions with finite energy). We also show that these solutions concentrate at a point in the limit, when we make $\varepsilon \to 0$. We use variational methods for this study. This work has financial support from FAPESP-2016/20798-5.

Joint work with Sérgio Henrique Monari Soares (ICMC/USP - São Carlos).

References

[1] C. O. Alves, S.H. M. Soares, *Existence and concentration of positive solutions for a class gradient systems*, NoDEA Nonlinear Differ. Equ. Appl. **12**,(2005), 437-457.

[2] R. Lehrer, *Existence of solutions for asymptotically linear systems in* $\mathbb{R}^N$, Elec. J. of Diff. Equ. **236**,Vol. 2013,(2013), 1-20.

[3] R. Lehrer, S. H. M. Soares, *Existence and concentration of positive solutions for a saturable coupled Schrödinger system*, preprint.

Existence of solution for a generalized quasilinear elliptic problem

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Abstract. We will establish existence of a solution to the elliptic quasilinear Schrödinger equation

$$-\operatorname{div}(g^2(u)\nabla u) + g(u)g'(u)|\nabla u|^2 + V(x)u = h(x,u), \quad x \in \mathbb{R}^N$$

where g, h, V are suitable smooth functions. The function g is asymptotically linear at infinity and, for each fixed $x \in \mathbb{R}^N$, the function $h(x,s)$ behaves like s at the origin and s^3 at infinity. In the proofs we apply variational methods.

Joint work with Furtado, M. F.(UnB) and Silva, E. D. (UFG).

References

[1] Furtado, M.F; Silva, E.D.; Silva, M. L., *Existence of solution for a generalized quasilinear elliptic problem*, JMP, vol 58 number 03, 2017

Local minimizers over the Nehari manifold for a class of concave-convex problems with sign changing nonlinearity

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Abstract. We study a p-Laplacian equation involving a parameter λ and a concave-convex nonlinearity containing a weight which can change sign. By using the Nehari manifold and the fibering method, we show the existence of two positive solutions on some interval $(0, \lambda^* + \varepsilon)$, where λ^* can be characterized variationally. We also study the asymptotic behavior of solutions when $\lambda \downarrow 0$.

Joint work with Kaye Oliverira Silva (Universidade Federal de Goiás)

References

[1] Ambrosetti, Antonio and Garcia Azorero, Jesus and Peral, Ireneo: *Multiplicity results for some nonlinear elliptic equations*, Journal of Functional Analysis 137 (1) (1996) 219–242.

[2] Ambrosetti, Antonio and Brezis, Haïm and Cerami, Giovanna: *Combined effects of concave and convex nonlinearities in some elliptic problems*, Journal of Functional Analysis 122 (2) (1994) 519–543.

[3] Il'yasov, Yavdat: *On extreme values of Nehari manifold method via nonlinear Rayleigh's quotient*, https://arxiv.org/abs/1509.08019 (2017).

Multiple positive solutions for a nonlocal problem from population genetics

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Abstract. We consider a nonlocal quasilinear elliptic problem under a flux boundary condition motivated by a model in population genetics. The reaction term is of strong Allee effect type, which takes place if the growth rate per capita is negative when the population density is small. Such term has two spatial dependent zeros that we do not require to be continuous functions. We prove the existence of two positive solutions for the problem when a parameter is large and that no positive solution can exist for small values of such parameter.

Joint work with Amanda A. F. Nunes (UFSCar).

Positive ground states for a class of superlinear (p, q)-Laplacian coupled systems involving Schrödinger equations

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Abstract. We study the existence of positive ground state solutions for the following class of (p, q)-Laplacian coupled systems

$$\begin{cases} -\Delta_p u + a(x)|u|^{p-2}u = f(u) + \alpha\lambda(x)|u|^{\alpha-2}u|v|^{\beta}, & x \in \mathbb{R}^N, \\ -\Delta_q v + b(x)|v|^{q-2}v = g(v) + \beta\lambda(x)|v|^{\beta-2}v|u|^{\alpha}, & x \in \mathbb{R}^N, \end{cases}$$

where $N \geq 3$ and $1 \leq p \leq q < N$. Here the coefficient $\lambda(x)$ of the coupling term is related with the potentials by the condition $|\lambda(x)| \leq \delta a(x)^{\alpha/p} b(x)^{\beta/q}$ where $\delta \in (0, 1)$ and $\alpha/p + \beta/q = 1$. We deal with periodic and asymptotically periodic potentials. The nonlinear terms $f(s)$, $g(s)$ are "superlinear" at 0 and at ∞ and are assumed without the well known Ambrosetti-Rabinowitz condition at infinity. Thus, we have established the existence of positive ground states solutions for a wider class of nonlinear terms and potentials. Our approach is variational and based on minimization technique over the Nehari manifold.

Joint work with Edcarlos D. Silva (UFG) and João Marcos do Ó (UFPB)

References

[1] L. Boccardo and D.G. de Figueiredo, 'Some remarks on a system of quasilinear elliptic equations', *NoDEA Nonlinear Differential Equations Appl.* 9 (2002) 309–323.

[2] J.M. do Ó and J.C. de Albuquerque, 'On coupled systems of nonlinear Schrödinger equations with critical exponential growth', *Applicable Analysis* (2017) 1–17. http://dx.doi.org/10.1080/00036811.2017.1296951

[3] J.M. do Ó and J.C. de Albuquerque, 'Positive ground sate of coupled systems of Schrödinger equations in $\mathbb{R}^2$ involving critical exponential growth', *Math. Meth. Appl. Sci.* (2017) 1–16. https://doi.org/10.1002/mma.4498

[4] L.A. Maia, E. Montefusco and B. Pellacci, 'Positive solutions for a weakly coupled nonlinear Schrödinger system', *J. Differential Equations* 229 (2006) 743–767.

[5] G.Q. Zhang, X.P. Liu and S.Y. Liu, 'Remarks on a class of quasilinear elliptic systems involving the (p, q)-Laplacian', *Electron. J. Differential Equations* 20 (2005) 1–10.

Fractional Kirchhoff problem with critical indefinite nonlinearity

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Abstract. We study the existence and multiplicity of positive solutions for a family of fractional Kirchhoff equations with critical nonlinearity of the form

$$M\left(\int_{\Omega}|(-\Delta)^{\frac{\alpha}{2}}u|^2dx\right)(-\Delta)^{\alpha}u = \lambda f(x)|u|^{q-2}u + |u|^{2^*_\alpha-2}u \text{ in } \Omega,\ u = 0 \text{ in } \mathbb{R}^n \setminus \Omega,$$

where $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain, $M(t) = a + \varepsilon t,\ a,\ \varepsilon > 0,\ 0 < \alpha < 1,\ 2\alpha < n < 4\alpha$ and $1 < q < 2$. Here $2^*_\alpha = 2n/(n-2\alpha)$ is the fractional critical Sobolev exponent, λ is a positive parameter and the coefficient $f(x)$ is a real valued continuous function which is allowed to change sign. By using a variational approach based on the idea of Nehari manifold technique, we combine effects of a sublinear and a superlinear term to prove the existence of at least two positive solutions for a suitable choice of parameter $\lambda > 0$ and sufficiently small $\varepsilon > 0$.

Joint work with Prof. João Marcos do Ó (UFPB, Brazil) and Prof. Xiaoming He (MUC, P. R. China).

References

[1] A. Ambrosetti, H. Brezis, and G. Cerami. Combined effects of concave and convex nonlinearities in some elliptic problems. *J. Funct. Anal.*, 122(2):519–543, 1994.

[2] D. Applebaum. *Lévy processes and stochastic calculus*, volume 116 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, second edition, 2009.

[3] G. Autuori, A. Fiscella, and P. Pucci. Stationary Kirchhoff problems involving a fractional elliptic operator and a critical nonlinearity. *Nonlinear Anal.*, 125:699–714, 2015.

[4] B. Barrios, E. Colorado, A. de Pablo, and U. Sánchez. On some critical problems for the fractional Laplacian operator. *J. Differential Equations*, 252(11):6133–6162, 2012.

[5] K. J. Brown and Y. Zhang. The Nehari manifold for a semilinear elliptic equation with a sign-changing weight function. *J. Differential Equations*, 193(2):481–499, 2003.

[6] X. Cabré and Y. Sire. Nonlinear equations for fractional Laplacians, I: Regularity, maximum principles, and Hamiltonian estimates. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 31(1):23–53, 2014.

[7] X. Cabré and J. Tan. Positive solutions of nonlinear problems involving the square root of the Laplacian. *Adv. Math.*, 224(5):2052–2093, 2010.

[8] L. Caffarelli. Non-local diffusions, drifts and games. In *Nonlinear partial differential equations*, volume 7 of *Abel Symp.*, pages 37–52. Springer, Heidelberg, 2012.

[9] L. Caffarelli and L. Silvestre. An extension problem related to the fractional Laplacian. *Comm. Partial Differential Equations*, 32(7-9):1245–1260, 2007.

[10] C.-Y. Chen, Y.-C. Kuo, and T.-F. Wu. The Nehari manifold for a Kirchhoff type problem involving sign-changing weight functions. *J. Differential Equations*, 250(4):1876–1908, 2011.

[11] A. Fiscella and E. Valdinoci. A critical Kirchhoff type problem involving a nonlocal operator. *Nonlinear Anal.*, 94:156–170, 2014.

[12] C.-Y. Lei, G.-S. Liu, and L.-T. Guo. Multiple positive solutions for a Kirchhoff type problem with a critical nonlinearity. *Nonlinear Anal. Real World Appl.*, 31:343–355, 2016.

[13] P.-L. Lions. The concentration-compactness principle in the calculus of variations. The limit case. I. *Rev. Mat. Iberoamericana*, 1(1):145–201, 1985.

[14] D. Naimen. The critical problem of Kirchhoff type elliptic equations in dimension four. *J. Differential Equations*, 257(4):1168–1193, 2014.

[15] P. Pucci and S. Saldi. Critical stationary Kirchhoff equations in $\mathbb{R}^N$ involving nonlocal operators. *Rev. Mat. Iberoam.*, 32(1):1–22, 2016.

[16] P. Pucci, M. Xiang, and B. Zhang. Existence and multiplicity of entire solutions for fractional p-Kirchhoff equations. *Adv. Nonlinear Anal.*, 5(1):27–55, 2016.

[17] R. Servadei and E. Valdinoci. The Brezis-Nirenberg result for the fractional Laplacian. *Trans. Amer. Math. Soc.*, 367(1):67–102, 2015.

[18] R. Servadei and E. Valdinoci. Fractional Laplacian equations with critical Sobolev exponent. *Rev. Mat. Complut.*, 28(3):655–676, 2015.

[19] X. Shang, J. Zhang, and Y. Yang. Positive solutions of nonhomogeneous fractional Laplacian problem with critical exponent. *Commun. Pure Appl. Anal.*, 13(2):567–584, 2014.

[20] L. Silvestre. Regularity of the obstacle problem for a fractional power of the Laplace operator. *Comm. Pure Appl. Math.*, 60(1):67–112, 2007.

[21] T.-F. Wu. On semilinear elliptic equations involving critical Sobolev exponents and sign-changing weight function. *Commun. Pure Appl. Anal.*, 7(2):383–405, 2008.

[22] T.-F. Wu. Multiple positive solutions for a class of concave-convex elliptic problems in $\mathbb{R}^N$ involving sign-changing weight. *J. Funct. Anal.*, 258(1):99–131, 2010.

[23] S. Yan, J. Yang, and X. Yu. Equations involving fractional Laplacian operator: compactness and application. *J. Funct. Anal.*, 269(1):47–79, 2015.

[24] J. Zhang, X. Liu, and H. Jiao, Multiplicity of positive solutions for a fractional Laplacian equations involving critical nonlinearity, *preprint available on arXiv:1502.02222v1.*

Existence of positive solutions of quasi-linear elliptic equations with gradient terms

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Abstract. We study the existence of non-negative solutions in the whole Euclidean space of coercive quasi-linear and fully nonlinear elliptic equations in the form

$$\Delta_p u = f(u) \pm g(|\nabla u|)$$

where

$$f \in C([0,\infty)), g \in C^{0,1}([0,\infty)) \text{ are strictly increasing with } f(0) = g(0) = 0.$$

We give conditions on f and g which guarantee the existence or absence of positive solutions of this problem in $\mathbb{R}^N$. These results represent a generalization to a result obtained for the case of the Laplacian operator, by Patricio Felmer, Alexander Quaas and Boyan Sirakov.

In the particular case of the problem with plus sign on the right-hand side we obtain generalized Keller-Osserman integral conditions. It turns out that different conditions are needed when $p \geq 2$ or $p \leq 2$.

References

[1] On solutions of $\Delta u = f(u)$. Keller, J. Communications on Pure and Applied Mathematics. Vol 10, N 4, pp 503-510, 1957

[2] On the inequality $\Delta u \geq f(u)$. Osserman, R. Pacific Journal of Mathematics. Vol 7, pp 1641-1647, 1957.

[3] The maximum principle. Pucci, P. and Serrin, J. Springer Science & Business Media Vol 73, 2007.

[4] Solvability of nonlinear elliptic equations with gradient terms. Felmer P., Quaas A., and Sirakov B. Journal of Differential Equations. Vol 11, pp 4327-4346, 2013.

Existence and multiplicity of positive solutions for a class of singular and nonlocal quasilinear problems

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Abstract. In these work, we will study the existence and multiplicity of positive solutions for the following problem involving the p-Laplacian operator

$$-\Delta_p u = \frac{\lambda}{(\int_\Omega g(u))^r}\Big(a(x)u^{-\delta} + b(x)u^\beta\Big) \text{ in } \Omega, \quad u > 0 \text{ in } \Omega \text{ and } u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a smooth and bounded domain, $\delta > 0$, $0 < \beta < p-1$, a and b are nonnegative measurable functions and g is a positive continuous function. Using technique of sub-supersolution, we will show that depending on δ, r and the behavior of g in the neighborhood of the origin and at the infinity, the given problem admits at least one, multiple or no solution.

Joint work with Carlos Santos (UnB).

References

[1] Alves, C. O, Covei, D., *Existence of solution for a class of nonlocal elliptic problem via sub-supersolution method*, Nonlinear Analysis: Real World Applications 23 (2015) 1-8.

[2] Arcoya, D., Leonori, T., Primo, A., *Existence of solutions for semilinear nonlocal elliptic problems via a Bolzano theorem*, Acta ApplMath 127 (2013), 87-104.

[3] Du, Y., *Bifurcation from infinity in a class a nonlocal elliptic problems*, Differential and Integral Equations 15 n 5 (2002), 587-606.

[4] García-Melián, J., Lis, J. S., *A boundary blow-up problem with a nonlocal reaction*, Nonlinear Analysis 75 (2012), 2774-2792.

A note on existence of a bound state when there is no ground state for a nonlinear scalar field equation

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Abstract. The aim of this work is to present a positive solution of a semilinear elliptic equation in $\mathbb{R}^N$ with non-autonomous non-linearities which are not necessarily pure-powers, nor homogeneous, and which are superlinear or asymptotically linear at infinity . The proof is variational combined with topological arguments.

Joint work with Liliane de Almeida Maia (UnB).

References

[1] Akhmediev, N. , Królinowski, W. and Snyder,, A. *Partially coherent solitons of variable shape* Physical Review Letters **81** , no. 21,(1998), 4632-4635.

[2] Bahri, A.; Li, Y. Y, *On a minimax procedure for the existence of a positive solution for certain scalar field* , Revista Mat. Iberoamericana. **6** (1990), n 1-2.

[3] Clapp, M. and Maia, L. A. , *A positive bound state for an asymptotically linear or superlinear Schrodinger equations* , J. Differential Equation. **260** (2016), 3173–3192.

[4] Évéquoz, G. and Weth, T. , *Entire solutions to nonlinear scalar field equations with indefinite linear part* , Adv. Nonlinear Studies. **12** (2012), 281-314.

[5] Maia, L. A.; Pellacci, B. , *Positive solutions for asymptotically linear problems in exterior domains*, Annali di Matematica Pura ed Applicata . (2016), 1-32.

A non periodic and asymptotically linear indefinite variational problem in $\mathbb{R}^N$

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Abstract. We consider the nonlinear Schrödinger equation

$$-\Delta u + V(x)u = f(u),$$

in $\mathbb{R}^N$, where V changes sign and f is an asymptotically linear function at infinity, with V non-periodic in x. The existence of a solution is established employing spectral theory, a classical linking theorem, interaction between translated solutions of the problem at infinity.

References

[1] A. Azzollini and A. Pomponio, On the Schrödinger equation in $\mathbb{R}^N$ under the effect of a general nonlinear term, *Indiana University Mathematics Journal*, **58** (2009), 1361–1378.

[2] D.G. Costa and H. Tehrani, Existence and multiplicity results for a class of Schrödinger equations with indefinite nonlinearities, *Adv. Difference Equation*, **8** (2003), 1319–1340.

[3] L. Jeanjean, K. Tanaka, A positive solution for an asymptotically linear elliptic problem on $\mathbb{R}^N$ autonomous at infinity, *ESAIM Control Optim. Calc. Var.*, **7** (2002), 597–614.

[4] L.A Maia, J.C. Oliveira Junior and R. Ruviaro, A non periodic and asymptotically linear indefinite variational problem in $\mathbb{R}^N$, *Indiana University Mathematics Journal*, **66**, issue 1 (2017).

[5] A. A. Pankov, Periodic nonlinear Schrödinger equation with application to photonic crystals, *Milan J. Math.*, **73** (2005), 259–287.

[6] A. Szulkin and T. Weth, Ground state solutions for some indefinite variational problems, *J. Func. Anal.*, **257** (2009), 3802–3822.

Some results for a class of quasilinear elliptic equations with singular nonlinearity

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Abstract. Motivated by recent works on the study of the equations that model some electrostatic MEMS devices, we study radially symmetric solutions for a general class of problems involving a second order elliptic operator in divergence form and a singular nonlinearity of inverse square type, with zero Dirichlet boundary condition. Our approach reach cases where the differential operator is, for instance, Laplacian, p-Laplacian and k-Hessian. This class of problems come up with a critical frame for the existence of solution, which can be expressed by the L^∞ norm of the nonlinearity. We present conditions over which we can assert regularity for the solution in the critical case. Moreover, we prove that whenever the critical solution is regular, there exists other solutions of mountain pass type close to the critical one. In addition, we use the Shooting Method to prove uniqueness of solutions when the nonlinearity has L^∞ norm close to 0.

Joint work with João Marcos Bezerra do Ó (UFPB).

References

[1] A. Ambrosetti and P. H. Rabinowitz, Dual variational methods in critical point theory and applications, *J. Functional Analysis*, 14:349–381, 1973.

[2] H. Brezis and L. Nirenberg, *H^1 versus C^1 local minimizers*, C. R. Acad. Sci. Paris Sér. I Math. **317(5)** (1993) 465–472 .

[3] D. Castorina, P. Esposito, and B. Sciunzi, *Degenerate elliptic equations with singular nonlinearities*, Calc. Var. Partial Differential Equations **34(3)** (2009) 279–306.

[4] P. Clément, D. G. de Figueiredo, and E. Mitidieri, *Quasilinear elliptic equations with critical exponents*, Topol. Methods Nonlinear Anal. **7(1)** (1996) 133–170.

[5] M. G. Crandall and P. H. Rabinowitz, *Bifurcation, perturbation of simple eigenvalues and linearized stability*, Arch. Rational Mech. Anal. **52** (1973) 161–180.

[6] M. G. Crandall and P. H. Rabinowitz, *Some continuation and variational methods for positive solutions of nonlinear elliptic eigenvalue problems*, Arch. Rational Mech. Anal. **58(3)** (1975) 207–218.

[7] J. Dávila, Singular solutions of semi-linear elliptic problems, In Handbook of differential equations: stationary partial differential equations. Vol. VI, Handb. Differ. Equ., pages 83–176. Elsevier/North-Holland, Amsterdam, 2008.

[8] J. M. do Ó and E. da Silva, *Quasilinear elliptic equations with singular nonlinearity. preprint.*

[9] P. Esposito, *Compactness of a nonlinear eigenvalue problem with a singular nonlinearity*, Commun. Contemp. Math. **10** (2008) 17–45.

[10] P. Esposito, N. Ghoussoub, and Y. Guo, *Compactness along the branch of semistable and unstable solutions for an elliptic problem with a singular nonlinearity*, Comm. Pure Appl. Math. **60** (2007) 1731–1768.

[11] P. Esposito, N. Ghoussoub, and Y. Guo, Mathematical analysis of partial differential equations modeling electrostatic MEMS, volume 20 of Courant Lecture Notes in Mathematics. Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2010.

[12] N. Ghoussoub and Y. Guo, *On the partial differential equations of electrostatic MEMS devices: stationary case*, SIAM J. Math. Anal. **38** (2006/07) 1423–1449 (electronic).

[13] Z. Guo and J. Wei, *Hausdorff dimension of ruptures for solutions of a semilinear elliptic equation with singular nonlinearity*, Manuscripta Math. **120** (2006) 193–209.

[14] Z. Guo and J. Wei, *Asymptotic behavior of touch-down solutions and global bifurcations for an elliptic problem with a singular nonlinearity*, Commun. Pure Appl. Anal. **7** (2008) 765–786.

[15] Z. Guo and J. Wei, *Infinitely many turning points for an elliptic problem with a singular nonlinearity*, J. Lond. Math. Soc. (2) **78** (2008), 21–35.

[16] J. Jacobsen and K. Schmitt, *The Liouville-Bratu-Gelfand problem for radial operators*, J. Differential Equations **184** (2002), 283–298.

[17] J. Jacobsen and K. Schmitt, Radial solutions of quasilinear elliptic differential equations. In Handbook of differential equations, pages 359–435. Elsevier/North-Holland, Amsterdam, 2004.

[18] D. D. Joseph and T. S. Lundgren, *Quasilinear Dirichlet problems driven by positive sources*, Arch. Rational Mech. Anal. **49** (1972/73) 241–269.

[19] J. P. Keener and H. B. Keller, *Positive solutions of convex nonlinear eigenvalue problems*, J. Differential Equations **16** (1974) 103–125.

[20] F. Mignot and J.-P. Puel, *Sur une classe de problèmes non linéaires avec non linéairité positive, croissante, convexe*, Comm. Partial Differential Equations **5** (1980), 791–836.

[21] V. Murthy and G. Stampacchia, *Boundary value problems for some degenerate-elliptic operators*, Ann. Mat. Pura Appl. (4), **80** (1968) 1–122.

[22] B. Opic and A. Kufner, Hardy-type inequalities, volume 219 of Pitman Research Notes in Mathematics Series, Longman Scientific & Technical, Harlow, 1990.

[23] J. A. Pelesko and D. H. Bernstein, Modeling MEMS and NEMS, Chapman & Hall/CRC, Boca Raton, FL, 2003.

On existence and concentration of solutions to a class of quasilinear problems involving the $1-$Laplace operator

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Abstract. In this work we use variational methods to prove results on existence and concentration of solutions to a problem in $\mathbb{R}^N$ involving the $1-$Laplacian operator. A thorough analysis on the energy functional defined in the space of functions of bounded variation $BV(\mathbb{R}^N)$ is necessary, where the lack of compactness is overcome by using the Concentration of Compactness Principle due to Lions.

Joint work with Claudianor O. Alves (UFCG)

Invertibility of nonsmooth mappings

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Abstract. In many corners of the analysis, invertibilty of functions plays an important role. Here we address our attention to the injectivity and invertibility of nonsmooth mappings by applying techniques of variational methods and nonsmooth analysis. We revisit the classical Hadamard-Levy theorem and the Plastock's results, and we also exploit the closed tied Jacobian and the Weak Marcus-Yamabe conjectures.

Joint work with Marcelo Montenegro (UNICAMP).

References

[1] Chamberland M.; Meisters, G. *A mountain pass to the Jacobian conjecture.* Canad. Math. Bull. **41** (1998) 442–451.

[2] Fernandes, A.. Gutiérrez C.; Rabanal R. *On local diffeomorphisms of* $\mathbb{R}^N$ *that are injective.* Qualitative Theory of Dynamical Systems **4** (2004) 255–262.

[3] Montenegro, M. S.; Presoto, A. E. *Invertibility of nonsmooth mappings.* Arkiv för Matematik. To appear.

Existence and uniqueness solution of $p(x)-$Laplacian operator for a singular-convex elliptic problem

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Abstract. In this paper we consider the existence, uniqueness and regularity of solutions in $W_{loc}^{1,p(x)}(\Omega)$ to an elliptic problem $-\Delta_{p(x)}u = a(x)u^{-\alpha(x)} + \lambda f(x,u)$ where $\alpha(x) > 1 - p_-$ and oscillate from positive to negative in multiple subregions of the interior and boundary of the domain. The proof of existence is based on a variant of the generalized Galerkin method. We also prove a Comparison Principle for functions in $W_{loc}^{1,p(x)}(\Omega)$ with a more general sense of zero-boundary value. Some of our results are new even for the Laplacian operator setting.

Joint work with Carlos A.P. Santos (University of Brasília)

References

[1] L. Boccardo and L. Orsina, Semilinear elliptic equations with singular nonlinearities, Calc. Var. Partial Differential Equations 37 (2010), 363-380.

[2] J. I. Daz and J. E. Saa, Existence et unicit de solutions positives pour certaines equations elliptiques quasilineaires, C. R. Acad. Sci. Paris I 305 (1978), 521-524.

[3] A. Canino, L. Montoro, B. Sciunzi, M. Squassina, Nonlocal problems with singular nonlinearity, Bull. Sci. Math. 141 (2017) 223-250.

Quasilinear elliptic equations with jumping nonlinearities in critical growth range

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Abstract. We consider the following critical Ambrosetti-Prodi problem for the p-Laplacian, given by

$$\begin{cases} -\Delta_p u &= \lambda|u|^{p-2}u + u_+^{p^*-1} + f(x) \quad \text{in} \quad \Omega \\ u &= 0 \quad \text{on} \quad \partial\Omega. \end{cases} \tag{11}$$

Here, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $\Omega \subset \mathbb{R}^N$ is a bounded domain, $1 < p < N$, $p^* = Np/(N-p)$, $\lambda < \lambda_1$ (λ_1 is the first eigenvalue for the p-Laplacian in Ω) and f is a forcing term in $L^\infty(\Omega)$.

We prove that, under suitable conditions on f, there exist at least two solutions for this problem, one of which is negative. This result extends previous achievments for the laplacian in the celebrated paper of De Figueiredo and Jianfu, *Critical superlinear Ambrosetti-Prodi problems*, Topol. Methods Nonlinear Anal. 14 (1999). There, the authors prove that there exist two solutions for problem (11) if $p = 2$ and $N > 6$.

We prove that a negative solution for this problem is obtained as a global minimum of a related p-linear problem and a second solution is found by considering a modified mountain pass geometry around the negative solution. We reach the same restriction to dimension N, found on the mentioned paper, when $p = 2$. Our main theorem reads

Theorem: Consider suitable conditions on f such that there exists a negative local minimum to the functional associated to problem (11). Then, there exists a second solution, provided $N > p(p^2 - p + 1)$ if $p > 2$ or $N > p^2 + p$ if $1 < p \leq 2$.

Joint work with Elisandra Gloss (UFPB) and Joo Marcos do (UFPB).

Mean field games with logistic efects

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Abstract. In its standard form, a mean-field game can be defined by a coupled system of equations, one Hamilton-Jacobi equation for the value function of agents and one Fokker-Planck equation for the density of agents. Traditionally, the latter equation is adjoint to the linearization of the former. Since the Fokker-Planck equation models a population dynamic, we introduce natural features such as immigration, birth, and non-linear death rates. Here we analyze a stationary mean-field game in one dimension, illustrating various techniques to obtain regularity of solutions in this class of systems. In particular we consider a logistic-type model for birth and death of the agents which is natural in problems where crowding affects the death rate of the agents. The introduction of these new terms requires a number of new ideas to obtain wellposedness. In a forthcoming publication we will address higher dimensional models.

Multiple solutions for an inclusion quasilinear problem with non-homogeneous boundary condition through Orlicz Sobolev spaces

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Abstract. In this work we study multiplicity of nontrivial solution for the following class of differential inclusion problems with non-homogeneous Neumann condition through Orlicz-Sobolev spaces,

$$\begin{cases} -div\big(\phi(|\nabla u|)\nabla u\big) + \phi(|u|)u \in \lambda \partial F(u) \text{ in } \Omega, \\ \frac{\partial u}{\partial \nu} \in \mu \partial G(u) \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a domain, $N \geq 2$ and $\partial F(u)$ is the generalized gradient of $F(u)$. The main tools used are variational methods for locally lipschitz functional and critical point theory.

Joint work with Rodrigo C. M. Nemer (UFCG).

Standing waves for a system of Schrödinger equations in $\mathbb{R}^N$ involving fractional laplacian

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Abstract. In this paper we study the existence of bound state solutions for stationary Schrödinger systems involving fractional laplacian of the form

$$\begin{cases} (-\Delta)^{\alpha/2} u + V(x)u = K(x)H_u(u,v) & \text{in} \quad \mathbb{R}^N, \\ (-\Delta)^{\alpha/2} v + V(x)v = K(x)H_v(u,v) & \text{in} \quad \mathbb{R}^N, \end{cases}$$

where $(-\Delta)^{\alpha/2}$ stands for the fractional laplacian with $\alpha \in (0,2)$, $N > \alpha$, V and K are bounded continuous nonnegative functions, $H(u,v)$ is a p-homogeneous function of class C^1 and $2 < p < 2^*_\alpha$. We give a special attention to the case when V may eventually vanishes. Our arguments are based on penalization techniques, variational methods and Moser iteration scheme.

Joint work with João Marcos do Ó (UFPB) and Olímpio H. Miyagaki (UFJF).

References

[1] L. A. Caffarelli, L. Silvestre, *An extension problem related to the fractional Laplacian*, Comm. Partial Differential Equations **32** (2007), no. 7-9, 1245-1260.

[2] E. Di Nezza, G. Palatucci, E. Valdinoci, *Hitchhiker's guide to the fractional Sobolev spaces*, Bull. Sci. math. **136** (2012) 521–573.

[3] J. M. do Ó, O. H. Miyagaki, C. R. Santana, *Standing waves for a system of nonlinear Schrödinger equations in* $\mathbb{R}^N$, Asymptotic Anal. **96** (2015) 351–372.

Existence of nontrivial solutions for the quasilinear schorödinger equation with subcritical growth

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Abstract. In this work we are interested in searching for non-trivial solutions to the quasilinear problem

$$-\Delta u + V(x)u - \Delta(u^2)u = g(x,u), \quad x \in \mathbb{R}^N \;\; u \in H^1(\mathbb{R}^{\mathbb{N}}) \tag{12}$$

where $N \geq 3$ and V is a positive potential. The nonlinearity $g(x,s)$ behaves like $K_0(x)s$ at the origin and like $K_\infty(x)|s|^p$, $1 \leq p \leq 3$, at infinity. Moreover, we consider the case where $g(x,s)$, is superlinear in infinity, that is,

$$\lim_{s \longrightarrow \infty} \frac{g(x,s)}{s^3} = \infty. \tag{13}$$

To obtain our results, we use Linking theorem introduced by Li and Willem in his famous article [2].

References

[1] Colin, M. and Jeanjean, L. - *Solutions for a quasilinear Schrödinger equation: a dual approach.*, Nonlinear Analysis: Theory, Methods & Applications, (2004), no. 2, 213–226.

[2] Li, S. J. and Willem, M. - *Applications of local linking to critical point theory*, Journal of Mathematical Analysis and Applications, 1995), no. 1, 6–32

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Analysis of branches of positive solutions for a quasilinear problem under the Nehari manifold method

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Abstract. This work concerns the application of the Nehari manifold method to the study of branches of positive solutions for the problem

$$-\Delta_p u = \lambda |u|^{p-2}u + f|u|^{\gamma-2}u,\ u \in W_0^{1,p}(\Omega),$$

where Δ_p is the p-Laplacian operator, f changes signs, λ is a real parameter and $1 < p < \gamma < p^*$. A special care is given to the extreme value λ^*, which is characterized variationally by

$$\lambda^* = \inf\left\{ \frac{\int |\nabla u|^p}{\int |u|^p},\ u \in W_0^{1,p}(\Omega),\ \int f|u|^\gamma \geq 0 \right\}.$$

The main result deals with the existence of two positive solutions when $\lambda \in (\lambda_1, \lambda^* + \varepsilon)$.

Joint work with Yavdat Il'yasov (UFA-Russia)

References

[1] S. Alama and G. Tarantello. *On semilinear elliptic equations with indefinite nonlinearities*, Cal. Var. and Part. Diff. Eq., **1** (4) (1993), 439–475.

[2] P. Drábek and S.I. Pohozaev. *Positive solution for the p-Laplacian: application of the fibering method*. Proc. Roy. Soc. Edinb. Sect. A., **127** (4) (1997), 703–726.

[3] Y, Il'yasov. *On extreme values of Nehari manifold method via nonlinear Rayleigh's quotient.* Topological Methods Nonlinear Analysis., **6** (2017), 1–31.

[4] Y, Il'yasov. and K, Silva, *On branches of positive solutions for p-Laplacian problems at the extreme value of Nehari manifold method*. arXiv:1704.02477 (2017).

[5] Z. Nehari, *On a class of nonlinear second-order differential equations*, Trans. Amer. Math. Soc. (1960) 101–123.

[6] T. Ouyang. *On the positive solutions of semilinear equations $\Delta u + \lambda u + hu^p = 0$ on the compact manifolds Part II*, Indiana Univ. Math. J., **40** (3) (1991), 1083–1141.

Nonlocal Schrödinger-Poisson systems with critical oscillatory growth

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Abstract. In this poster, we are concerned with the existence of nontrivial solutions for the following class of nonlinear fractional Schrödinger–Poisson system

$$\begin{aligned} (-\Delta)^s u + a(x)u + \lambda K(x)\phi u = f(x,u) + g(x,u) \quad &\text{in} \quad \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = K(x)u^2 \quad &\text{in} \quad \mathbb{R}^3, \end{aligned}$$

where $0 < s < 1$, $0 < \alpha < 1$, $2\alpha + 4s \geq 3$, $\lambda > 0$. We consider the presence of potential $a(x)$ which may change sign and nonlinearities $f(x,t)$ and $g(x,t)$ having oscillatory subcritical and critical growth respectively. The potential $K(x) \geq 0$ belongs to a suitable Lebesgue space and the methods used here are based on the concentration of compactness principle of P. L. Lions and on the profile decomposition for weak convergence in Hilbert spaces due to K. Tintarev and K.-H Fieseler.

References

[1] A. Ambrosetti, On Schrödinger-Poisson systems, *Milan J. Math.* **76** (2008) 257–274.

[2] J. M. do Ó and D. Ferraz, Concentration-compactness at the mountain pass level for nonlocal Schrödinger equations https://arxiv.org/abs/1610.04724 (2016).

[3] J. M. do Ó and D. Ferraz, Concentration-compactness principle for nonlocal scalar field equations with critical growth, *J. Math. Anal. Appl.* **449** (2017) 1189–1228.

[4] X. Shang, J. Zhang and Y. Yang, On fractional Schrödinger equation in $\mathbb{R}^N$ with critical growth, *J. Math. Phys.* **54** (2013) 121502, 20.

On a Kirchhoff type equation with critical exponential growth in $\mathbb{R}^2$

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Abstract. In this work we estudy the Kirchhoff problem

$$m\left(\int_{\mathbb{R}^2} (|\nabla u|^2 + b(x)u^2)\ \mathrm{d}x\right)(-\Delta u + b(x)u) = A(x)f(u), \quad x \in \mathbb{R}^2,$$

where $m : [0, \infty) \to (0, \infty)$ and $f : \mathbb{R} \to [0, \infty)$ are continuous functions and $b, A \in L^\infty_{\text{loc}}(\mathbb{R}^2)$. The potential b can be negative or vanish on sets with positive measure and the function f has critical growth in the sense of Trudinger-Moser inequality. We consider suitable assumptions on b, A and f that allow us to treat this problem variationally in the space

$$H := \left\{u \in W^{1,2}(\mathbb{R}^2) : \int_{\mathbb{R}^2} b(x)u^2\ \mathrm{d}x < \infty\right\}\ .$$

Applying the Mountain Pass Theorem, we establish the existence of a nonnegative ground state solution.

Joint work with Marcelo F. Furtado (University of Brasília).

Necessary and sufficient conditions for existence of Blow-up solutions for elliptic problems in Orlicz-Sobolev spaces

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Abstract. This paper is principally devoted to revisit the remarkable works of Keller and Osserman and generalize some previous results related to the those for the class of quasilinear elliptic problem

$$\begin{cases} \operatorname{div}(\phi(|\nabla u|)\nabla u) = a(x)f(u) \quad \text{in } \Omega, \\ u \geq 0 \ \text{ in } \Omega, \quad u = \infty \text{ on } \partial\Omega, \end{cases}$$

where either $\Omega \subset \mathbb{R}^N$ with $N \geq 1$ is a smooth bounded domain or $\Omega = \mathbb{R}^N$. The function ϕ includes special cases appearing in mathematical models in nonlinear elasticity, plasticity, generalized Newtonian fluids, and in quantum physics. The proofs are based on comparison principle, variational methods and topological arguments on the Orlicz-Sobolev spaces.

Joint work with Carlos A. Santos (UnB) and Jefferson A. Santos (UFCG).

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