Formal Models for Security and Authenticity

Escola de Verão - Matemática - UnB

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Roteiro

- Lógica Clássica Proposicional

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- Lógica Clássica Proposicional
- Lógica Clássica de 1^a Ordem

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- Lógicas Modais

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- Lógica Epistêmica Multi-Agente

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- Lógicas Modais
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- Formalismos Lógicos/ Algébricos para Segurança e Autenticação

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- Lógicos $\Longrightarrow$ Dedução
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- Algébrica $\Longrightarrow$ Equivalência entre Expressões
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Lógica Clássica Proposicional

- **Poder Expressão:**

- **Problema 1:** Dada uma fórmula φ com comprimento n e uma valoração v para os símbolos proposicionais. Qual a complexidade de se calcular o valor para a atribuição **v**?

$$O(n)$$

- **Problema 2:** Dada fórmula φ com comprimento n e m símbolos proposicionais. Verificar se existe alguma valoração que satisfaz φ.

$$O(2^m.n) \text{ – } \textbf{NP-Completo}$$

Lógica Clássica de 1^a Ordem

- **Poder Expressão:**

- **Problema 1 (Satisfabilidade):** Dada uma fórmula φ uma estrutura $\mathcal{E}$. Qual a complexidade de calcular o valor verdade de φ?

Indecidível

- **Problema 2 (Validade):** Dada fórmula φ. Verificar se existe alguma estrutura $\mathcal{E}$ satisfaz φ.

Indecidível

Poder Expressão: Est. Finitas

- **Problema 1 (Satisfabilidade/Verificação de Modelos):** Dada uma fórmula φ uma estrutura **finita** $\mathcal{E}$. Qual a complexidade de calcular valor verdade de φ?

PSPACE-Completo

- **Problema 2 (Validade):** Dada fórmula φ. Verificar se existe alguma estrutura **finita** $\mathcal{E}$ que satisfaz φ.

Indecidível

Lógicas Modais

- **Linguagem Modal**
- Conjunto de Proposições atômicas

$$\varphi ::= p \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \rightarrow \varphi_2 \mid \neg\varphi \mid \Box\varphi \mid \Diamond\varphi$$

- $\Box p$: todo mundo que eu vejo marcou p como V
- $\Diamond p$: alguém que eu vejo marcou p como V

Semântica Modal

- Mundos Possíveis/Estados
- Fórmulas são avaliadas em grafos $F = (W, R)$
  W é um conjunto não-vazio de *estados* e
  R é uma relação binária em W

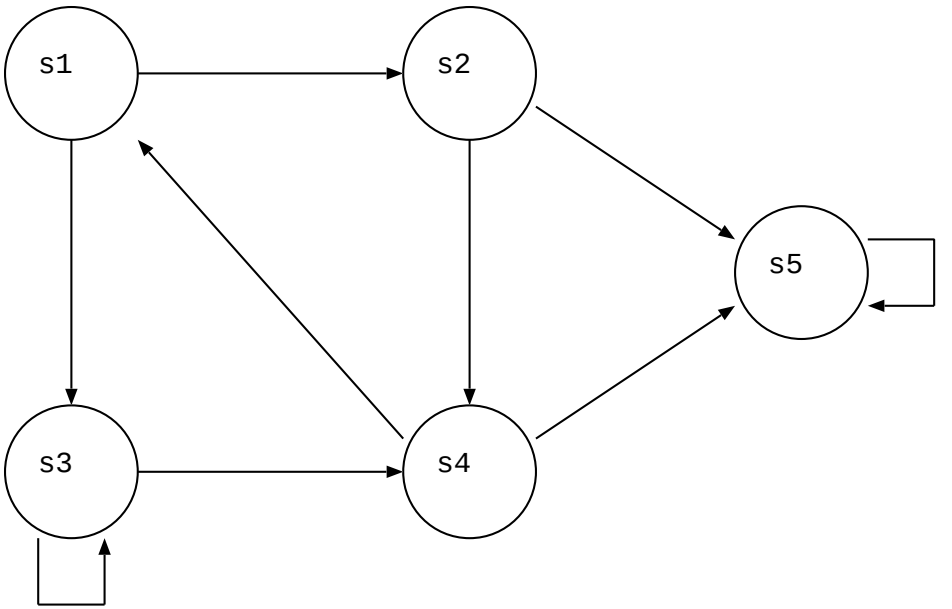
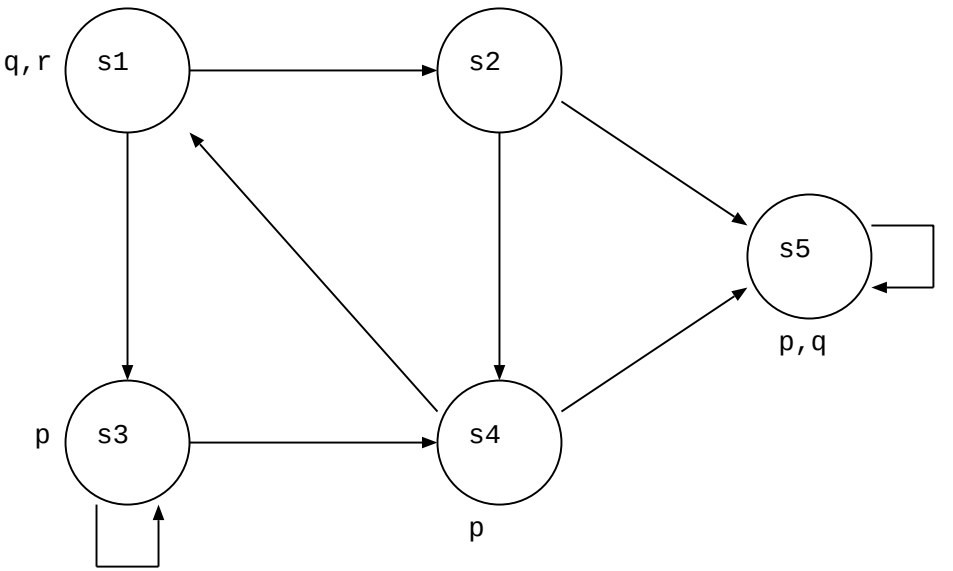


Figura 1: Exemplo de um Frame.

Semântica Modal

- Fórmulas são avaliadas em grafos $F = (W, R)$ rotulados com proposições atômicas
- *modelo* $M = (F, V)$ onde
  $F = (W, R)$ é um *frame* e
  V é uma função associa a cada p o conjunto de estados nos quais p é verdadeiro



Satisfação

- $M, w \Vdash p$ sse $w \in V(p)(\forall p \in \Phi)$
- $M, w \Vdash \neg\varphi$ iff $M, w \nVdash \varphi$,
- $M, w \Vdash \varphi \rightarrow \varphi'$ sse $M, w \nVdash \varphi$ **ou** $M, w \Vdash \varphi'$
- $M, w \Vdash \varphi \wedge \varphi'$ sse $M, w \Vdash \varphi$ **e** $M, w \Vdash \varphi'$
- $M, w \Vdash \varphi \vee \varphi'$ sse $M, w \Vdash \varphi$ **ou** $M, w \Vdash \varphi'$
- $M, w \Vdash \Box\varphi$ sse para todo $w' \in W$ **se** wRw' **implica** $M, w' \Vdash \varphi$
- $M, w \Vdash \Diamond\varphi$ sse existe $w' \in W$, wRw' **e** $M, w' \Vdash \varphi$

Modalidades $\times$ Lógica Modal

- Modalidades $\surd$
  - Tempo
  - Conhecimento: Saber e Acreditar
  - Obrigação e Permição

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- **Modelo**: SIM

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- **Resposta**: SIM e NÃO
- **Modelo**: SIM
- **Validade**: NÃO - Nem Sempre

Comlexidade e Expressividade

- **Verificação de Modelos** $O(|\varphi| \times (|W| + |R|))$

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- **P** $\subseteq$ **NP** $\subseteq$ **PSPACE** $\subseteq$ **EXPTIME**

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- **Validade**: para **S5** é **NP**-Completo.
- **P** $\subseteq$ **NP** $\subseteq$ **PSPACE** $\subseteq$ **EXPTIME**
- **Validade**: **EXPTIME**-Completo,
  - Lógica Dinâmica Proposicional PDL
  - Lógica Epistêmica Multi-agente (c/ Conhecimento Comum)
  - CTL - Computation Tree Logic (Tempotal)
  - μ-Calculus (Menor Ponto Fixo)

Lóg. Epistêmica Multi-agentes

- Conjunto Finito de Agentes $G = a, b, c, ...$
- Duas modalidades para cada agente
- $K_a\varphi$ - $a$na saber φ
- $B_a\varphi$ - $a$na acredita em φ
- $B_a\varphi = \neg K_a \neg\varphi$
- Modalidades de Grupos:
- $E_G\varphi$ - o grupo G sabe φ
- $C_G\varphi$ - φ é de conhecimento comum do grupo G

Modelando Conhecimento

- **Agentes**: **a**na e **b**eto

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- Carta contendo a informação:

  $p =$ "ana ganhou R$ 1,00"

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- envelope lacrado e sobre a mesa
- O que ana e beto sabem?

Modelando Conhecimento

- Dois estados possíveis para ana e beto

$$\underline{s_1} - a, b - s_2$$

Modelando Conhecimento

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- $s_1 =$ "ana ganhou R$ 1,00"$\Rightarrow\ p$

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- ana e beto não sabem se estão em s_1 ou s_2

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- $K_a \neg K_b p$ - ana sabe que beto não sabe p

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- $K_a \neg K_b p$ - ana sabe que beto não sabe p
- $E_G \neg K_b p$ - o grupo sabe que beto não sabe p

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- $E_G \neg K_b p$ - o grupo sabe que beto não sabe p
- $C_G \neg K_b p$ - conhecimento comum q beto ñ sabe p

$\underline{s_1} - a, b - s_2$

Lógica Epistêmica - Linguagem

- Alfabeto
  - Φ conj. contável de símbolos prop.,
  - $\mathcal{A}$ conjunto finito de agentes,
  - $\neg$ e $\wedge$ conectivos booleanos,
  - K_a uma modalidade para cada agente a,
  - C_G uma modalidade para cada agente a.
- **Linguagem**

$$\varphi ::= p \mid \top \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid K_a\varphi \mid C_G\varphi$$

onde $p \in \Phi$, $a \in \mathcal{A}$.

Semântica

- Um *frame* é um par $F = (W, \sim_a)$ onde
  - W é um conjunto não-vazio de *estados*;
  - $\sim_a$ é uma relação binária para cada agente a
    - Reflexiva
    - Transitiva
    - Simétrica
  - $\sim_G = \bigcup_{a \in G} \sim_a$
  - $\sim_G^\star$ fecho reflexivo transitivo de $\sim_G$
- Um *modelo* é um par $M = (F, V)$ onde
  - $F = (W, R)$ é um *frame* e
  - V é uma função que faz corresponder a todo símb. prop. $p \in \Phi$ o conjunto de estados nos quais p é satisfeito, i.e., $V : \Phi \longmapsto Pow(W)$.

Semântica

- Dada uma estrutura $\mathcal{M} = \langle S, \sim_a, V \rangle$

$$
\begin{array}{lll}
\mathcal{M}, s \models p & iff & s \in V(p) \\
\mathcal{M}, s \models \neg\phi & iff & \mathcal{M}, s \not\models \phi \\
\mathcal{M}, s \models \phi \wedge \psi & iff & \mathcal{M}, s \models \phi \text{ e } \mathcal{M}, s \models \psi \\
\mathcal{M}, s \models K_a\phi & iff & \forall t : s \sim_a t \text{ implica } \mathcal{M}, t \models \phi \\
\mathcal{M}, s \models C_G\phi & iff & \forall t : s \sim_G^\star t \text{ implica } \mathcal{M}, t \models \phi
\end{array}
$$

Axiomatização

- **Axiomas**

  1. Tautologias proposicionais,
  2. $K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$,
  3. $K_a\varphi \to \varphi$,
  4. $K_a\varphi \to K_aK_a\varphi \qquad (+\, introspection)$,
  5. $\neg K_a\varphi \to K_a\neg K_a\varphi \qquad (-\, introspection)$,
  6. $C_G(\varphi \to \psi) \to (C_G\varphi \to C_G\psi)$,
  7. $C_G\varphi \to (\varphi \wedge E_GC_G\varphi)$
  8. $C_G(\varphi \to E_G\varphi) \to (\varphi \to C_G\varphi)$ Indução

- **Regras de inferência**
  M.P. $\varphi, \varphi \to \psi/\psi$ U.G. $\varphi/K_a\varphi$

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- **Modelo Finito**

Lógica Modal

- **Porque**

Lógica Modal

- **Porque**
- **Onde**

Lógica Modal

- **Porque**
- **Onde**
- **Quando**

Lógica Modal: Porque

- **Decidíveis**

Lógica Modal: Porque

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Lógica Modal: Porque

- **Decidíveis**
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- **Verificação de Modelos: Polinomial**

Lógica Modal: Onde

- **Grafos Rotulados**

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- **Fecho Transitivo**: Iteração, Conhecimento Comum, Until
- **Ponto Fixo**: Menor e Maior

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  Lóg. Modais correspondem ao fragmento de LCPO invariante por Bissimulação

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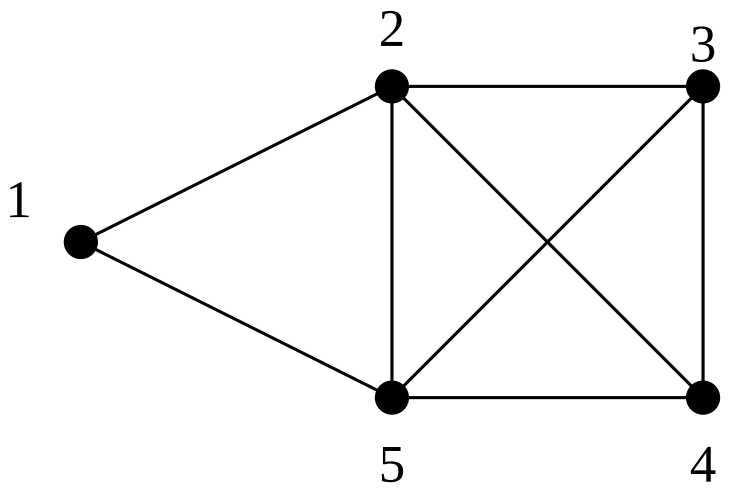
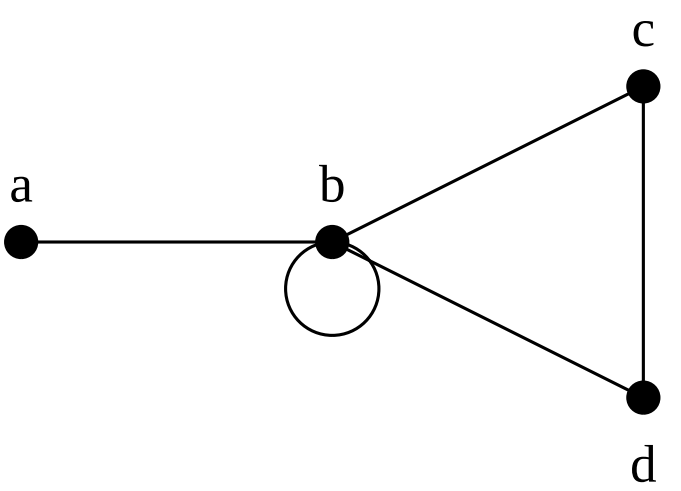
- LM boa para falar de propriedades que são **Invariantes** por bissimulação

Lógica Modal: Quando Não

- **Hamiltonian Graphs**:

  Existe um ciclo que percorre todos os nós exatamente uma vez.





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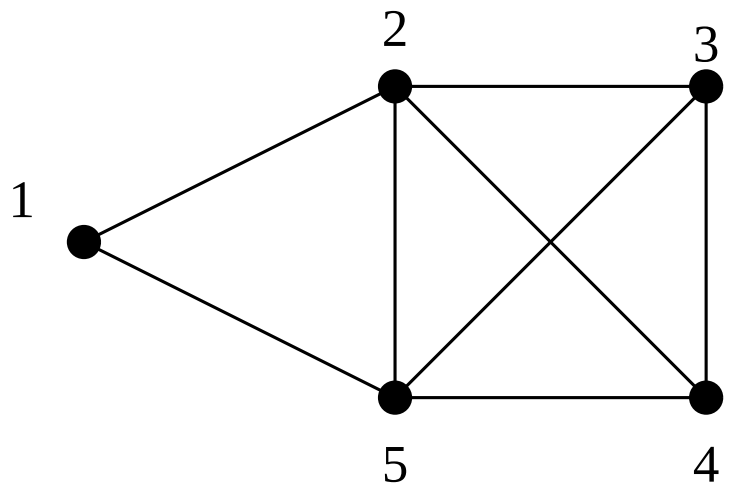

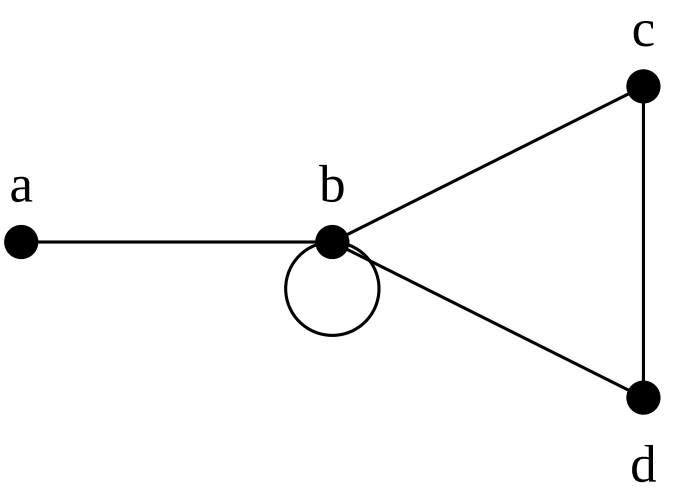

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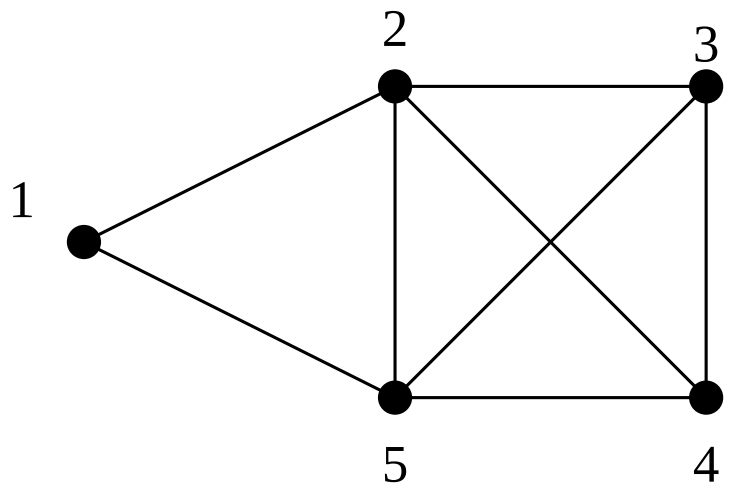

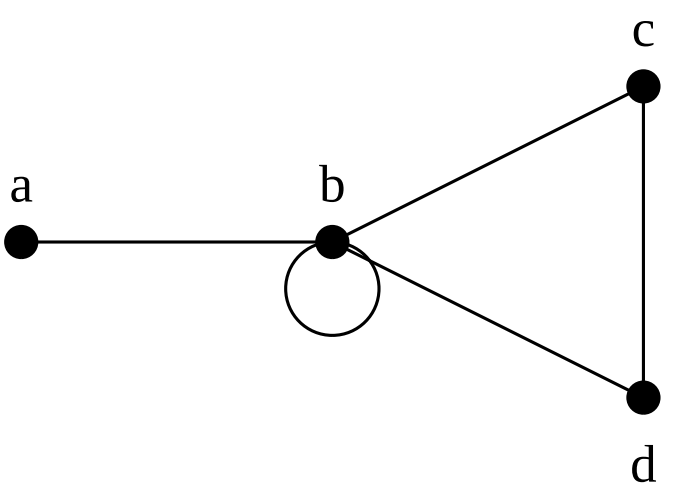

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- **O que fazer???**: Lógicas Hibridas, Memory Logics , etc...

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- **Model: Public Key Protocols**

- encryption function E_X (public)
- decryption function D_X (known only by user X)
- Requirements:

- $D_X E_X(M) = M$
- for any user Y knowing $E_X(M)$ does not reveal anything about M

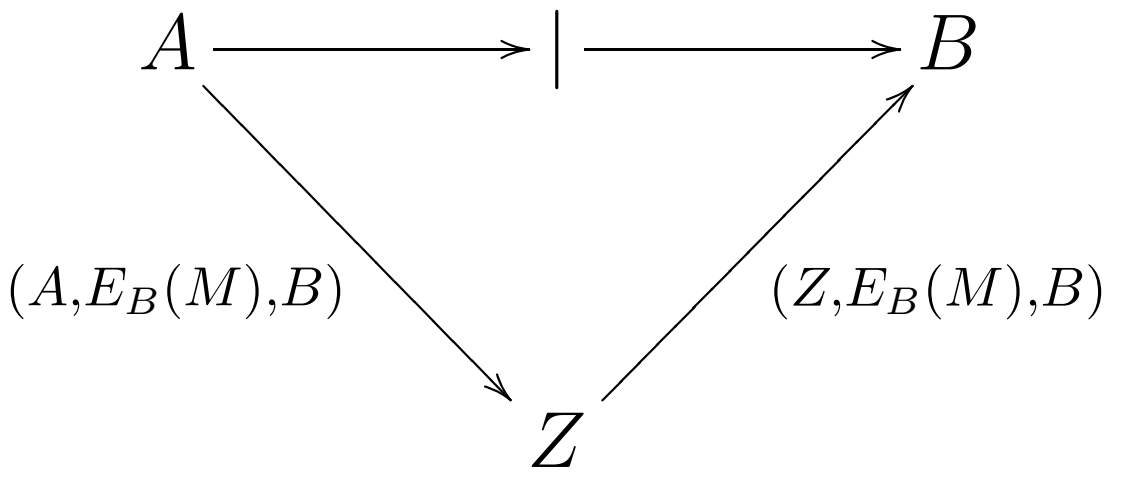
Example 1: Dolev & Yao Model

A sends msg M to B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

Intruder Z intercepts the message sent from A to B

Intruder Z sends message $(Z, E_B(M), B)$ to B



Example 1: Dolev & Yao Model

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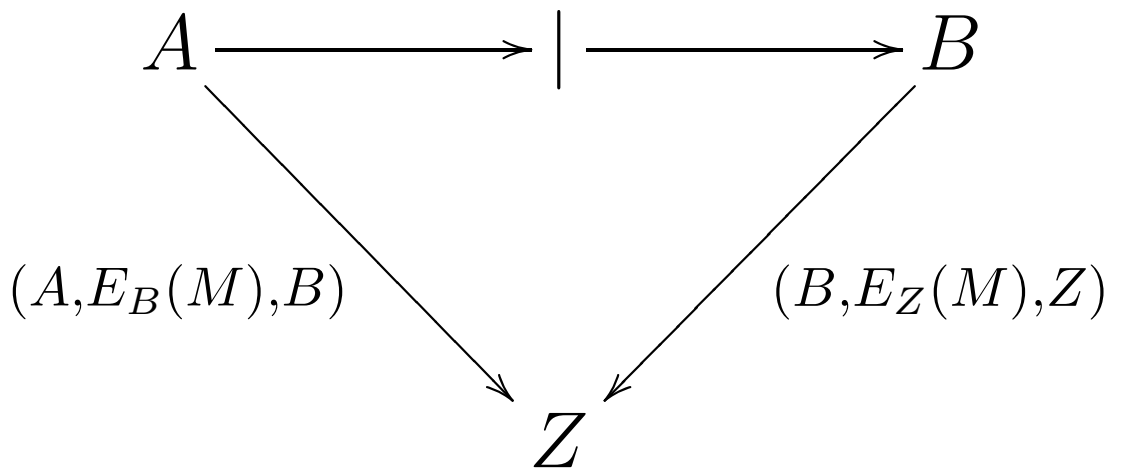


Intruder Z decodes $E_Z(M)$ and obtains M

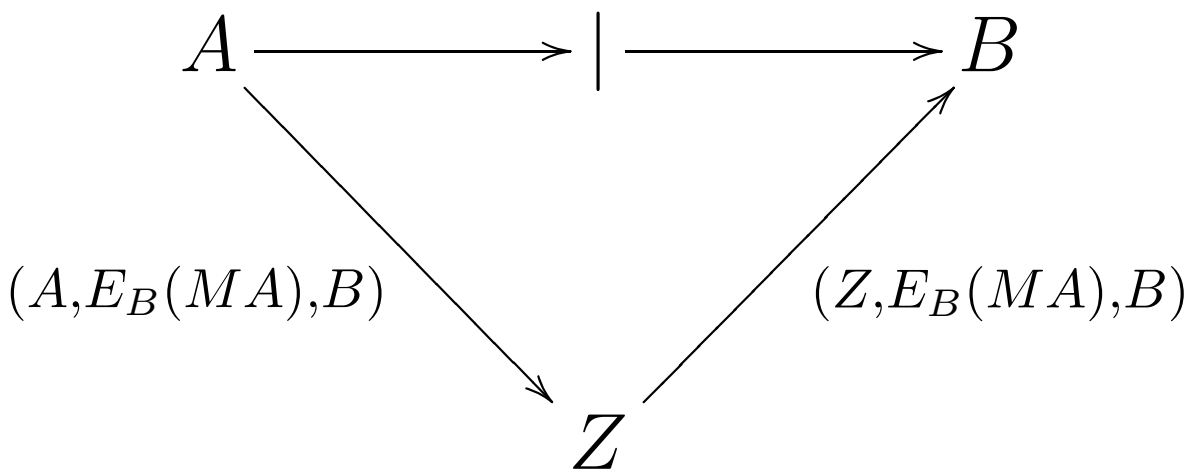
Example 2: Dolev & Yao Model

A sends msg MA to B and B replies to the user that is encrypted with the message M and not to the sender

$$A \longrightarrow (A, E_B(MA), B) \longrightarrow B$$

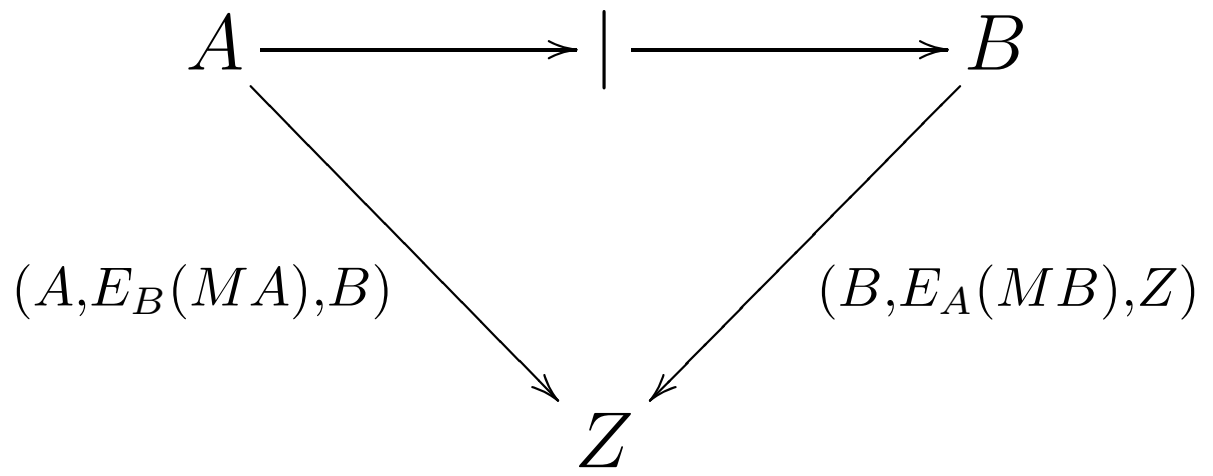
Intruder Z intercepts the message sent from A to B

Intruder Z sends message $(Z, E_B(MA), B)$ to B



Example 2: Dolev & Yao Model

B sends message $(B, E_A(MB), Z)$ to Z



Intruder Z **cannot** decode $E_A(MB)$ to obtain M

It can be proved that this protocol in secure against arbitrary behaviour of the intruder.

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- Let T be all the information Z has.

Rules : Dolev & Yao Model

Reflexivity

$$\frac{M \in T}{T \vdash M}$$

Encryption

$$\frac{T \vdash K \quad T \vdash M}{T \vdash \{M\}_K}$$

Decryption

$$\frac{T \vdash \{M\}_K \quad T \vdash K}{T \vdash M}$$

Pair – Composition

$$\frac{T \vdash M \quad T \vdash N}{T \vdash (M, N)}$$

Pair – Decomposition

$$\frac{T \vdash (M, N)}{T \vdash M} \qquad \frac{T \vdash (M, N)}{T \vdash N}$$

Proving Example 1

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2.4 Applying pair composition to 2.1 and 2.3:
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4.5 Applying Decryption rule to 4.3 and 4.4 we obtain: $T \vdash M$

Expressions Equivalence

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Formal Encryption

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Formal Encryption

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- Two pieces of data “look the same”
- $\Box$ represents a ciphertext that an attacker cannot decrypt.
- If $E \equiv \Box$ means that an attacker cannot decrypt E.
- Expressions are all the information that the Intruder has intercepted.

Language

- **Expressions**:

$$E ::= i \mid K \mid (E_1, E_2) \mid \{E\}_K \mid \Box$$

Language

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Language

- **Expressions**:

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- where $i \in \{0, 1\}$ - Bits (Messages)
- $K \in \{K_1, \cdots\}$ - Keys
- $\Box$ - undecryptable

Rules

- Rules are as in Dolev and Yao. Let M and N be expressions

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- if $M \vdash (N_1, N_2)$, then $M \vdash N_1$ and $M \vdash N_2$

Equivalence

- **Paterns**: Intuitively, it is an expression that may have some parts that an attacker cannot decrypt.
- Let T be a set of Keys
- Function $p(E,T) \mapsto E$

$$
\begin{array}{lcll}
p(K,T) & = & K & \\
p(i,T) & = & i & \\
p((M,N),T) & = & (p(M,T),p(N,T)) & \\
p(\{M\}_K,T) & = & \{p(M,T)\}_K & if\ K \in T \\
 & = & \square & otherwise
\end{array}
$$

- $patern(M) = p(M,\{K \mid M \vdash K\})$

Equivalence

- $M \equiv N$ iff $patern(M) = patern(N)$
- Example: let $T = \{K_2, K_3\}$
- $\{(\{0\}_{K_1}, \{1\}_{K_2})\}_{K_3} \equiv \{(\Box, \{1\}_{K_2})\}_{K_3}$
- $pattern(\{(\{0\}_{K_1}, \{1\}_{K_2})\}_{K_3}) =$
  $pattern(\{(\Box, \{1\}_{K_2})\}_{K_3})$
- $p(\{(\{0\}_{K_1}, \{1\}_{K_2})\}_{K_3}, T) =$
- $\{p((\{0\}_{K_1}, \{1\}_{K_2}), T)\}_{K_3} =$
- $\{((p(\{0\}_{K_1}, T), p(\{1\}_{K_2}, T))\}_{K_3} =$
- $\{(\Box, \{p(1, T)\}_{K_2})\}_{K_3} =$
- $\{(\Box, \{1\}_{K_2})\}_{K_3}$

Equivalence

- Example (Cont.): let $T = \{K_2, K_3\}$
- $\{(\{0\}_{K_1}, \{1\}_{K_2})\}_{K_3} \equiv \{(\Box, \{1\}_{K_2})\}_{K_3}$
- $pattern(\{(\{0\}_{K_1}, \{1\}_{K_2})\}_{K_3}) = pattern(\{(\Box, \{1\}_{K_2})\}_{K_3})$
- $M \equiv N$ iff $patern(M) = patern(N)$
- $M \cong N$ iff $M \equiv N\sigma$, σ is a renaing function
- If we can prove $M \cong \Box$, then M is **undecryptable**

Spi Calculus

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- Similar to the π-Calculus
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- Terms are processes
- Equivalence Relation: Bisimulation

Language - Spi Calculus

- The language of the Spi-Calculus is very similar to the Pi-Caculus.
- In the standard Pi-Calculus terms are only names.

Terms:

$L, M, N ::=$		Terms
	n	name
	(M,N)	pair
	0	zero
	suc(M)	successor
	x	variable
	$\{M\}_N$	shared-key encryption

Processes - Spi Calculus

$P, Q, R ::=$		Processes
	$\bar{M}(N).P$	output
	$M(x).P$	input
	$P \mid Q$	parallel compos
	$(\nu)P$	restriction
	$!P$	replication
	$[M\ is\ N]P$	match
	$\mathbf{0}$	nul
	$let\ (x, y) = M\ in\ P$	pair splitting
	$case\ M\ of\ 0 :\ suc(x) : Q$	integer case
	$case\ L\ of\ \{x\}_N\ in\ P$	shared-key decr

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- Process $case\ M\ of\ 0 :\ suc(x) : Q$ behaves as P if term M is 0, as $Q[N/x]$ if M is $suc(N)$. Otherwise, the process is stuck.
- Process $case\ L\ of\ \{x\}_N\ in\ P$ attempts to decrypt the term L with the key N. If L is a ciphertext of the form $\{M\}_N$, then the process behaves as $P[M/x]$. Otherwise, the process is stuck.

Example - Spi Calculus

- Example with key establishment

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- Wide Mouthed Frog

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- 3 agents: A, B and S (server)

Example - Spi Calculus

- Example with key establishment
- Wide Mouthed Frog
- 3 agents: A, B and S (server)
- A and B share keys K_{AS} and K_{SB} repectively with server S

Example - Spi Calculus

- Protocol:
- A creates a key K_{AB}
- A send key K_{AB} under encription K_{AS}
- S decript the mesage and send key K_{AB} under encription K_{SB}
- A send mesage M under encription K_{AB}

$$A \xrightarrow{\{K_{AB}\}_{K_{AS}}} S \xrightarrow{\{K_{AB}\}_{K_{SB}}} B$$

Example - Spi Calculus

- **Protocol**

$$A(M) = (\nu K_{AB})(\bar{c}_{AS}\langle\{K_{AB}\}_{K_{AS}}\rangle.(\bar{c}_{AB}\langle\{M\}_{K_{AB}}\rangle))$$
$$S = c_{AS}(x).case\ x\ of\ \{y\}_{K_{AS}}\ in\ \bar{c}_{SB}\langle\{y\}_{K_{SB}}\rangle$$
$$B = c_{SB}(x).case\ x\ of\ \{y\}_{K_{SB}}\ in\ c_{AB}(z).case\ z\ of\ \{w\}_y\ in\ F(w)$$
$$Inst(M) = (\nu K_{AS})(\nu K_{SB})(A(M) \mid S \mid B)$$

- Since all communication is protected by encryption, communication can take place through public channels: c_{AS}, c_{SB} and c_{AB}

Secrecy - Spi Calculus

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Secrecy - Spi Calculus

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- **Secrecy**: The message M cannot be read in transit from A to B: if F does not reveal M , then the whole protocol does not reveal M.
- The secrecy property can be stated in terms of equivalences: if $F(M) \simeq F(M'$ for all M and M' , then $Inst(M) \simeq Inst(M')$. This means that if $F(M)$ is indistinguishable from $F(M')$, then the protocol with message M is indistinguishable from the protocol with message M' .

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- $Inst_{espc}$ is $Inst$ substituting B by B_{espec}

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- $Inst_{espc}$ is $Inst$ substituting B by B_{espec}
- **Authenticity**: The run of the protocol is not affected by any message that an intruder can send to B,

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Summary - Spi Calculus

- Equivalence between processes: Bisimulation $\simeq$
- **Secrecy**: $Inst(M) \simeq Inst(M')$, if $F(M) \simeq F(M')$ for all M and M'
- **Secrecy**: The message M cannot be read in transit from A to B
- **Authenticity**: $Inst(M) \simeq Inst_{spec}(M)$, for all M
- **Authenticity**: The run of the protocol is not affected by any message that an intruder can send to B

BAN Logic

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  M. Burrows, M. Abadi, and R. Needham. ACM Transactions on Computer Systems, 8:18–36, 1990.

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Notation - BAN Logic

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- K, ranges over encryption keys.

Sintaxe- BAN Logic

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- $\langle X \rangle_Y$: This represent X combined with the formula Y. In implementations, X is simply concatenated with the password Y.

Logical Postulates - BAN Logic

Message-meaning: rules for interpretation of messages.

For shared keys

$$\frac{P \,\mathbf{believes}\, Q \overset{K}{\leftrightarrow} P, \quad P \,\mathbf{sees}\, \{X\}_K}{P \,\mathbf{believes}\, Q \,\mathbf{said}\, X}$$

For public keys

$$\frac{P \,\mathbf{believes}\, \overset{K}{\mapsto} Q, \quad P \,\mathbf{sees}\, \{X\}_{K^{-1}}}{P \,\mathbf{believes}\, Q \,\mathbf{said}\, X}$$

Logical Postulates - BAN Logic

Message-meaning: rules for interpretation of messages.

For shared secrets

$$\frac{P\,\mathbf{believes}\,Q \overset{Y}{\rightleftharpoons} P, \quad P\,\mathbf{sees}\,\langle X\rangle_Y}{P\,\mathbf{believes}\,Q\,\mathbf{said}\,X}$$

Logical Postulates - BAN Logic

Jurisdiction:

$$\frac{P \text{ believes } Q \text{ controls } X, \quad P \text{ believes } Q \text{ believes } X}{P \text{ believes } X}$$

Logical Postulates - BAN Logic

Principal sees:

$$\frac{P \,\mathbf{sees}\, (X, Y)}{P \,\mathbf{sees}\, X} \qquad \frac{P \,\mathbf{sees}\, \langle X \rangle_Y}{P \,\mathbf{sees}\, X}$$

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Logical Postulates - BAN Logic

Fresh:

$$\frac{P \,\mathbf{believes} \quad \mathbf{fresh}(X)}{P \,\mathbf{believes} \quad \mathbf{fresh}\,(X, Y)}$$

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$$\frac{P\,\mathbf{believes}\,\forall V_1\ \ldots\ V_n.(Q\,\mathbf{controls}\,X)}{P\,\mathbf{believes}\,Q'\,\mathbf{controls}\,X'}$$

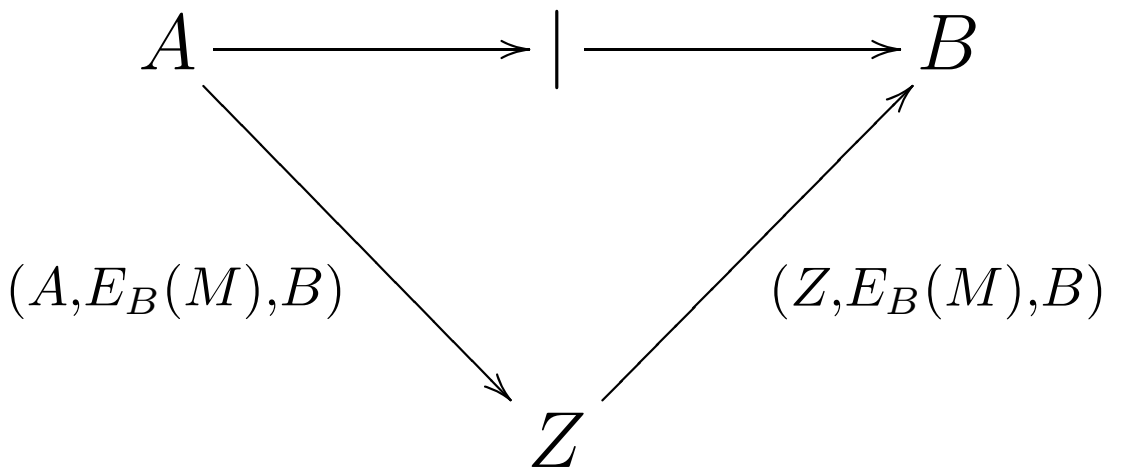
Example 1: Dolev & Yao Model

A sends msg M to B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

Intruder Z intercepts the message sent from A to B

Intruder Z sends message $(Z, E_B(M), B)$ to B



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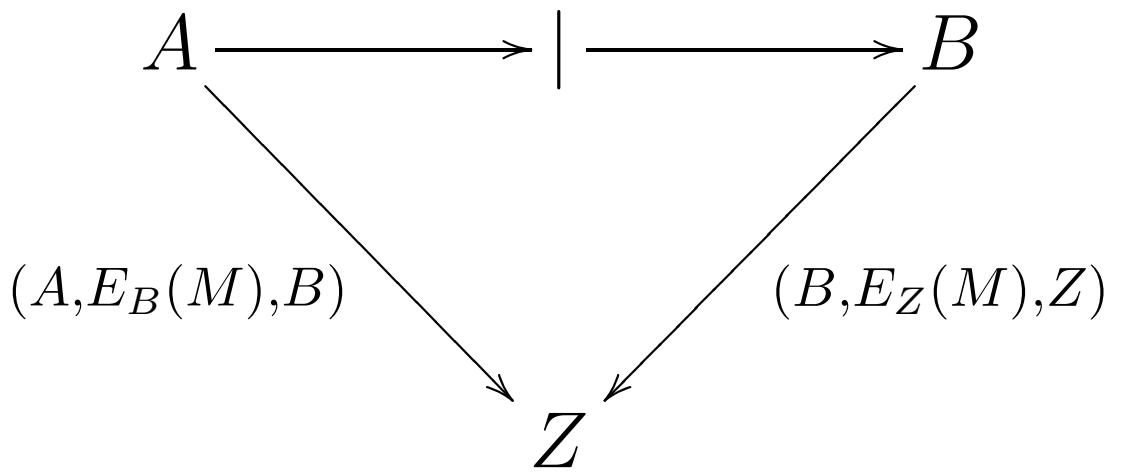

Intruder Z decodes $E_Z(M)$ and obtains M

Example - BAN Logic

- $m_1 : A \longrightarrow B : \{m\}_{K_B}$
- $m_2 : Z \longrightarrow B : \{m\}_{K_B}$
- $m_3 : B \longrightarrow Z : \{m\}_{K_Z}$
- B believes $A \overset{K_B}{\leftrightarrow} B$
- Z believes $B \overset{K_Z}{\leftrightarrow} Z$
- $m_1 : Z$ sees $\{m\}_{K_B}$
- $m_2 : B$ sees $\{m\}_{K_B}$
- B sees m $\quad rule\ principal\ sees$
- $m_3 : Z$ sees $\{m\}_{K_Z}$
- Z sees m $\quad rule\ principal\ sees$

Dolev/Yao Epistemic Logic

- Dolev/Yao Multi-Agent Epistemic Logic $\mathbf{S5_{DY}}$

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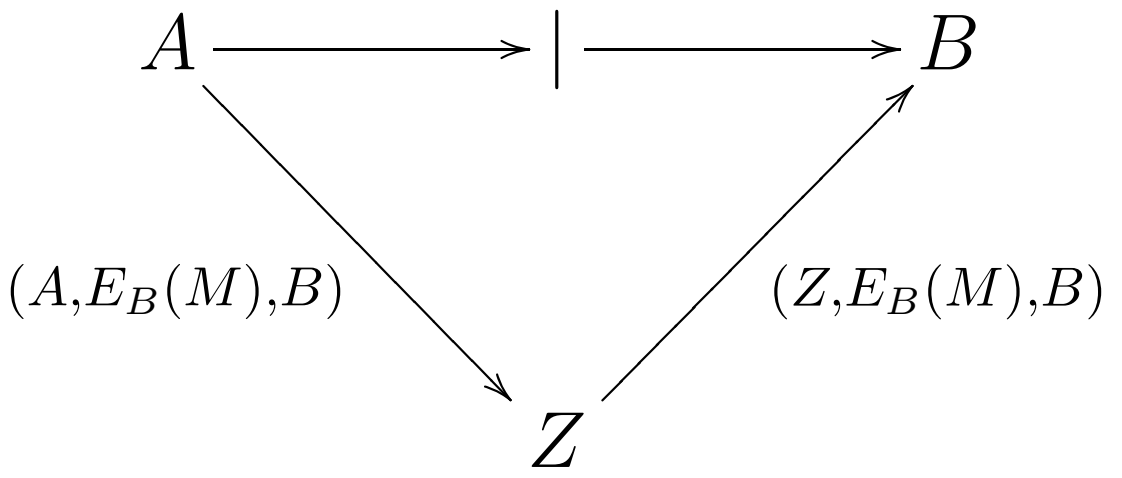

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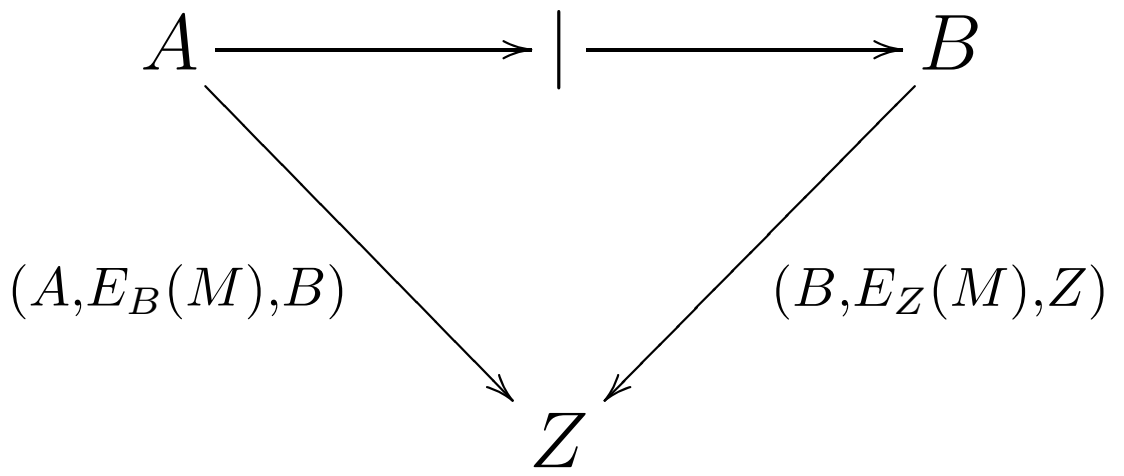

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- **Initial Knowledge**:

$$KB_0 = \{K_A k_{AB},\ K_B k_{AB},\ K_B k_{BZ},\ K_Z k_{BZ},\ K_A m\}$$

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$$\Big\downarrow send_{AB}(\{m\}_{k_{AB}})$$

$$- \ - \ -$$

$$\Big\downarrow Z\ intecepts$$

$$KB_1 := KB_0 \cup K_Z\{m\}_{k_{AB}}$$

$$\Big\downarrow send_{ZB}(\{m\}_{k_{AB}})$$

$$KB_2 := KB_1 \cup K_B\{m\}_{k_{AB}}$$

$$K_B m \quad ax.\ 7.$$

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$$K_B m \quad ax.\ 7.$$

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$$\downarrow send_{BZ}(\{m\}_{k_{BZ}})$$

$$KB_3 := KB_2 \cup K_Z\{m\}_{k_{BZ}}$$

$$K_Z m \quad ax.7$$

Intruder Z knows M

Example 2: Dolev & Yao Model

A sends msg MA to B and B replies to the user that is encrypted with the message M and not to the sender

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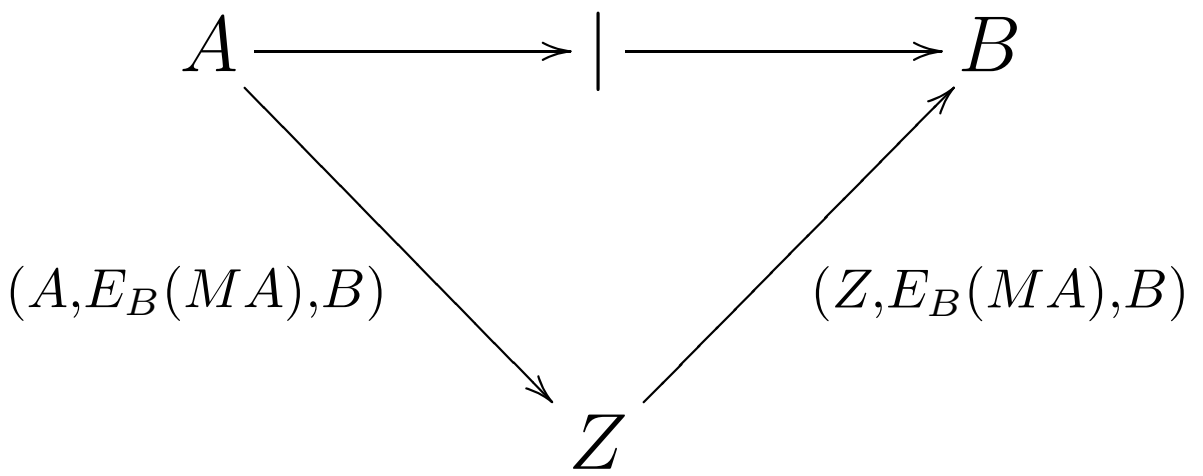

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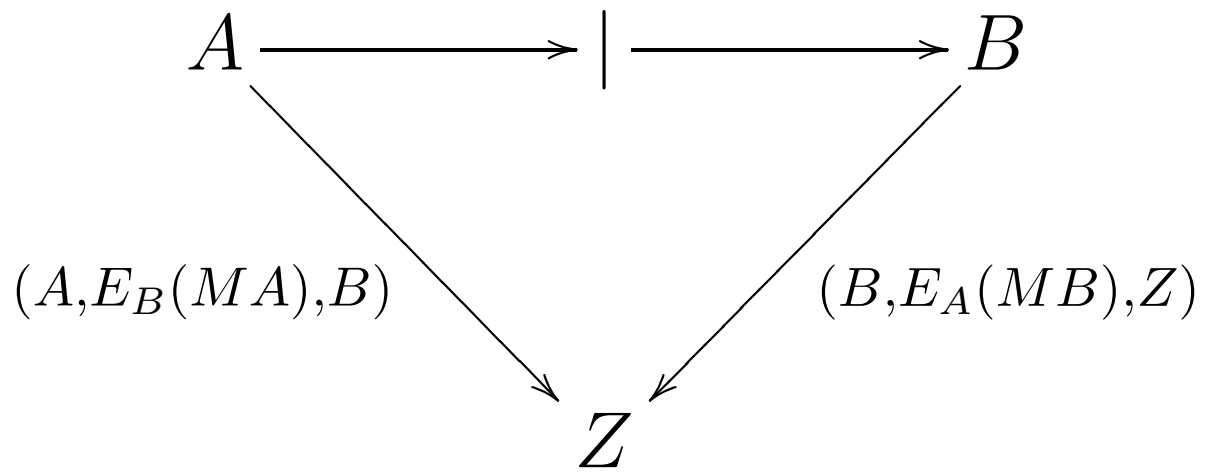

Intruder Z **cannot** decode $E_A(MB)$ to obtain M

It can be proved that this protocol in secure against arbitrary behaviour of the intruder.

Proving Example 2

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$$KB_0 = \{K_A k_{AB},\ K_B k_{AB},\ K_B k_{BZ},\ K_Z k_{BZ},\ K_A m\}$$

$$KB_0 \vdash K_A(k_{AB}, m)$$

$$KB_0 \vdash K_A\{(k_{AB}, m)\}_{k_{AB}} \quad ax.\ 6$$

$$\Big\downarrow {\scriptstyle send_{AB}(\{(k_{AB},m)\}_{k_{AB}})}$$

$$- \ - \ -$$

$$\Big\downarrow {\scriptstyle Z\ intecepts}$$

$$KB_1 := KB_0 \cup K_Z\{(k_{AB}, m)\}_{k_{AB}}$$

$$\Big\downarrow {\scriptstyle send_{ZB}(\{(k_{AB},m)\}_{k_{AB}})}$$

$$KB_2 := KB_1 \cup K_B\{(k_{AB}, m)\}_{k_{AB}}$$

Proving Example 2

$$KB_2 := KB_1 \cup K_B\{(k_{AB}, m)\}_{k_{AB}}$$

$$K_B(k_{AB}, m) \quad ax.\ 7.$$

$$K_B m \quad ax.\ 8.$$

$$K_B\{(k_{AB}, m)\}_{k_{AB}} \quad ax.\ 6.$$

$$\downarrow send_{BZ}(\{(k_{AB},m)\}_{k_{AB}})$$

$$KB_3 := KB_2 \cup K_Z\{(k_{AB}, m)\}_{k_{AB}}$$

$$KB_3 \nvdash K_Z m$$

More Examples

- Third example of the original article of Dolev & Yao

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- Kerberos Protocol

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- Andrew Secure RPC Handshake Protocol

Adding Actions

- Adding Actions to $\mathbf{S5_{DY}}$

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- Action Dolev/Yao Multi-Agent Epistemic Logic $\mathbf{S5^{A}_{DY}}$

  axiom: $K_A m \rightarrow [send_{AB}(M)]K_B m$???????

Future Works

- Adding Common Knowledge to $\mathbf{S5_{DY}}$

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