$Q[\alpha, \beta, \gamma] = \sup \sum_{App} \epsilon_i \epsilon_j$, where the sup is over all possible approaching waves that can be created by re-orderings of waves up to time T, when quadratic effects alone are included.

Here L measures the total variation, P the supnorm, and Q decreases at interaction by third order terms that bound the increase in L and P in the full nonlinear system. It is important that the functionals are defined entirely in terms of the quadratic model; and since we are maximizing over all possible re-orderings of waves consistent with a bounded wave speed, the nonlinearity of the wave speeds does not affect this analysis – from the point of view of the analysis of the functionals, we might as well assume all wave speeds are constant at the maximum possible actual value. If the functionals are finite on a finite sequence of waves then we say that the functionals are *bounded*; and if the functionals evaluated on nearby sequences of waves are bounded by the total variation of the difference between the wave sequences, then we say that the functionals are *continuous*. We prove that all of these functionals are bounded and continuous for the class of Euler type systems studied here. We then prove that, across an interaction diamond in the full nonlinear problem, the following estimates hold, c.f. [2]:

$$L(J_+) - L(J_-) \leq K_L S D, \tag{3.21}$$

$$P(J_+) - P(J_-) \leq K_P S D, \tag{3.22}$$

$$Q(J_+) - Q(J_-) \leq -(1 - K_Q S) D, \tag{3.23}$$

and

$$F(J_+) - F(J_-) \leq K_F S D, \tag{3.24}$$

where $F(J) = L(J) + KQ(J)$ for K sufficiently large. These estimates enable us to follow the proof strategy outlined in [13] to obtain the exponential growth in L when the supnorm is sufficiently small.

As a final comment, see [1] for an interesting new regularity result for solutions generated by Glimm's method that is related to the problem of the stability of solutions in the d-norm.