RESEARCH ON DIFFERENTIAL GEOMETRY IN BRAZIL*

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Some friends of mine insisted that I should present a personal account of research developments in Differential Geometry in Brazil at the 10th School of Differential Geometry. They said, "you have been an eyewitness to the better part of these developments, and you should give an account of that before you leave us". Even though I don't feel I'm in any great hurry "to leave", I feel myself under the obligation to write this report which, added to others, might be of interest. Thus, I will talk a little about the history of Differential Geometry in Brazil.

Initially, I should point out that, as there are many things that go by the name of Differential Geometry, I will need to be more specific. Differential Geometry here shall refer to a structure which allows us to speak of curvature (which "measures" how much a given structure differs locally from a standard structure.) This includes, for example, Riemannian Geometry, where the standard structure is Euclidean Geometry, Conformal Geometry and Projective Differential Geometry. But it does not include Symplectic Geometry, (according to Darboux's Theorem, two neighborhoods of symplectic manifolds are always equivalent; thus there are no local invariants) the pseudo Lie groups and the theory of singularities. Not that these subjects are not important; they are, but I should restrict myself to what I can speak of with confidence.

I will divide my historical account into three periods: The first, which I shall call the Prehistory of Geometry, and to which I was not an eyewitness, extends from 1800 to 1957. The second, the Beginning of the History of Differential Geometry, extends from 1957 to 1970. And, finally, there is the Research Consolidation Period, which extends from 1970 to 1983. The end of this last period is entirely arbitrary, but I believe the facts to be sufficiently registered from this date onwards and my account to be less significant. Furthermore, the making of history entails a certain distancing, and as a friend of mine

*Translated from Portuguese by Barbara B. F. Melo once wisely stated: "Anything less than ten years from now is not history - it's politics".

The Prehistorical Period contains very little of Differential Geometry. From a general perspective of Mathematics, though, various names stand out in this period. Among the most well-known we find, in order of their birth dates, the following:

Joaquim Gomes de Souza
1829–1863

Otto de Alencar
1874–1912

Amoroso Costa
1885–1928

Lélio Gama
1892–1981

Teodoro Ramos
1896–1936

A brief account of each of these scholars follows, in addition to which greater details may be found in the references given.

Joaquim Gomes de Souza, "Souzinha", from the State of Maranhão, came to Rio de Janeiro to study at the Military School. He gave up his military career to study medicine. His great passion, however, was the study of natural sciences and he was convinced, as were so many others of his day, that sooner or later all natural phenomena would be reduced to Mathematics. He spent some time in Paris completing his studies in mathematics. The objective of his book, Mélanges de Calcul Integral (there is a copy at the IMPA library,) is to obtain a general method for the solution of partial differential equations. He employed methods not entirely rigorous and it is not clear exactly how much of his work would remain if submitted to a careful scrutiny; as far as I know, it was never put to such a test.

A conference on Joaquim Gomes de Souza was given by Teodoro Ramos, and later published in the Anais da Academia Brasileira de Ciências. 1 (1929), 164-170.

Otto de Alencar was a professor at the Polytechnical School in Rio de Janeiro. He was one of the first Brazilian mathematicians to rebel against the tyranny of Augusto Comte's philosophy on the teaching of the Sciences in Brazil (especially that of Mathematics). As we will mention further on, such influence led to everyone's ignoring and underplaying almost all of the mathematical advances of the 19th Century. There is a paper on Otto de Alencar's mathematical work, written by Amoroso Costa and published in his remarkable book As idéias fundamentais da Matemática e outros ensaios (Fundamental ideas in Mathematics and other essays). Editorial Grijalbo Ltda., Editora da Universidade de São Paulo, 1971, 67-86. We will come back to Otto de Alencar in a little while.

Amoroso Costa was also a professor at the Polytechnical School in Rio de Janeiro. Greatly influenced by Otto de Alencar, he was one of the greatest disseminators of some of the most recent mathematical ideas of his time, which he synthesized in his previously mentioned book. One can also find two short biographies of Amoroso Costa in this book.

Lélio Gama was also a professor at the Polytechnical School. Director at IMPA since its founding in 1952 until 1965, his influence on the new generation of mathematicians was fundamental. I gave a brief summary of his mathematical achievements on the occasion of his 80th birthday: this work was published in the Boletim da Sociedade Brasileira de Matemática, 3 (1972), 160-164. I will speak again of Lélio Gama further on.

Teodoro Ramos was a student at the Polytechnical School in Rio de Janeiro, where he was a contemporary and friend of Lélio Gama. After a while, he transferred to São Paulo in the position of Professor. I was unable to obtain any study that analyzed his work.

Among the important facts in the Prehistorical period, we should mention the founding of the Faculdade de Filosofia, Ciências e Letras (School of Philosophy, Sciences, and Languages) at the University of São Paulo - USP in 1934, and of the Conselho Nacional de Pesquisas, CNPq (the National Research Council) in 1951.

It is high time we made a survey of what was done during this period in Brazilian mathematics. I would just like to mention here what the Brazilian contribution to Differential Geometry was during this time. I was able to find only two papers: one of Otto de Alencar, published in 1898 in the Revista da Escola Politécnica (vol. 3, 137-144), and another of Lélio Gama, presented as thesis to the Astronomy Chair at the Polytechnical School in 1929.

Otto de Alencar's work refers to the minimal surface of Riemann. Such a surface is generated by the movement of a circle (of variable radius) in such a way that, in this movement, the plane of the circle is kept parallel to the initial plane (Figure 1). It appeared in a posthumous work of Riemann published in 1869 (Complete Works, translated into French. Gauthier-Villars, 1898). The difficulty lies in obtaining the expression of the variable radius and the equations of the curve described by the center of the circle in such a way that the resulting surface is minimal. Otto de Alencar solves the problem by means of elliptic functions, and writes the implicit surface equation in terms of the Jacobi function. The development is simpler and more elegant than the cited work of Riemann (which was actually written by Hatendorff, based on a manuscript of Riemann, which had no text - just formulas.) Alencar's work is not very original (it is an exercise in elliptic functions) but it is certainly interesting.

When I say that it is an exercise in elliptic functions, this is no discredit. It is worthwhile to point out that elliptic functions, which today make up the curriculum of Mathematics, were almost unheard of in Brazil at that time. Not that they were new to the world of mathematics; they had been introduced in the first half of the 19th Century (around 1830), and by the end of the century were a fairly common instrument and, depending on the issue, indispensable. Nevertheless, the influence of Augusto Comte's philosophy in Brazil retarded the introduction of various new ideas in Mathematics. Comte stated that Mathematics was ready and finished around the end of the 18th Century, and that all that was left to do was apply it. This dogmatic attitude was accepted at the Military School and in the few Engineering Schools, which were, at the time, the places where Mathematics was cultivated in Brazil. It was only very slowly, by means of a few clear-headed thinkers, as the ones mentioned above, that the new ideas in Mathematics began to be established in Brazil.

Lélio Gama's work is entitled "Study of geodesic lines". The author's objective is to study geodesic triangles (that is, those whose sides are geodesic) on the terrestrial spheroid, which is a rotation ellipsoid with small eccentricity. More precisely, the objective is to compare the angles of a given geodesic triangle on a spheroid to the corresponding angles of a plane triangle whose sides are equal to the given triangle. According to a tradition in Differential Geometry initiated by Gauss, Lélio Gama generalizes the problem and studies the geodesics of any convex surface. Developing in series the equations of such geodesics up to the fourth power by two different processes, and equating the coefficients, he obtains a certain amount of information on the curvature and torsion of the geodesic lines of any convex surface. These results are then applied to the case of a rotation ellipsoid, and finally to the spheroid to solve the proposed problem. It is a beautiful work in classical differential geometry.

Perhaps it would be convenient to mention that Gauss's fundamental work -General Investigations of Curved Surfaces, Raven Press, New York, 1965; translated into English from the original "Disquisitiones generales circa superficies curvas", (Goettingen, 1827) - which marked the coming of age of Differential Geometry, was also motivated by a problem in Geodesy. Gauss was placed in charge of a geodesic survey of a region in Germany. This work involved measuring triangles on the surface of the Earth, which led him to reflect on the influence the shape of the Earth had on these measurements. Being a mathematician, he generalized the question to any surface and obtained, for small geodesic triangles, what we call today the Gauss-Bonnet theorem, which is the most important result in Classical Differential Geometry. (The famous theorem egregium, which also composed this work, can be obtained as a corollary of the Gauss-Bonnet theorem).

Riemann's Minimal Surface

Following Lélio Gama's work, there is a gap in Brazilian Differential Geometry. Actually, it was very difficult to do mathematical research of any kind at that time. Scarcity of journals, scientific isolation and lack of social incentive were but a few of the difficulties that had to be met by those who would have liked to do research. Lélio Gama himself, probably the most successful mathematical researcher of his day, gave up on Mathematics upon perceiving that it was impossible to keep up with what was going on abroad, and dedicated the rest of his life to Astronomy. Here - he would say - I have the Southern Hemisphere; Mathematics is the same everywhere, but the Southern Hemisphere, this no one can take from me.

The gap that opened up after Lélio Gama's work lasted until March 1957 when Alexandre M. Rodrigues from USP presented his doctoral thesis in Chicago, under the supervision of S.S. Chern, the greatest differential geometer of the time. The work dealt with characteristic classes in complex homogeneous spaces and was published in the Boletim da Sociedade Matemática de São Paulo in 1958 (vol. 10 [1955], 67-86, MR22 No. 4082.)

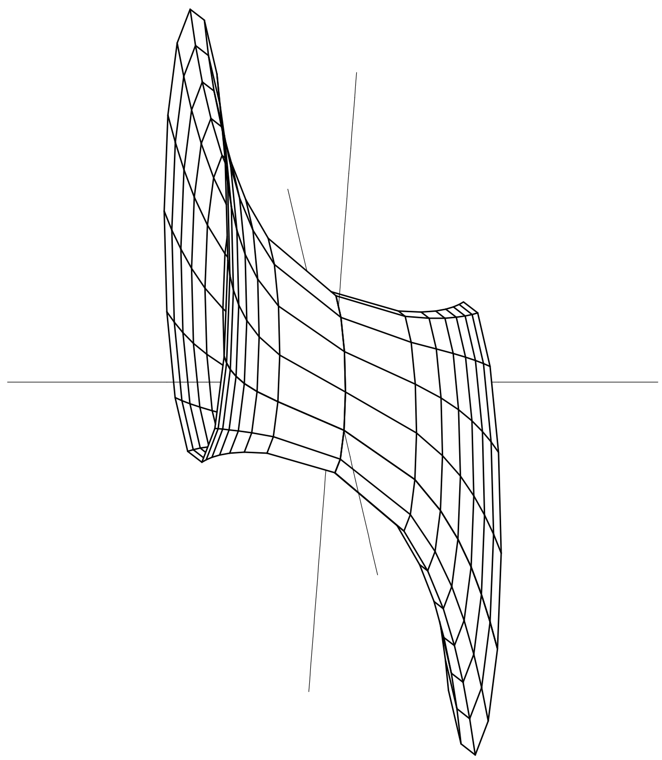

Figure 1: rieman.eps

In July of that same year the 1st Brazilian Mathematics Colloquium was held in Poços de Caldas, a gathering whose influence would be decisive to the future of Brazilian mathematics. Thence sprung a movement that would substantially enlarge mathematical research in Brazil. We have chosen the year 1957 to mark the beginning of the second period in the history of Differential Geometry in Brazil.

In October 1960, Léo Amaral and I went to study with S.S. Chern in Berkeley, California. I have described my recollections as a student of Chern in an article which appeared in the 12th issue of Matemática Universitária, 1990, 16-21 [1]. In January 1963, I finished my Doctorate with a thesis on the relationship between curvature and topology, which was published in the Annals of Mathematics 81 (1965), 1-14. Léo Amaral concluded his Doctorate in May 1964 with a thesis on hypersurfaces of non-Euclidean spaces; as far as I know, this thesis was never published.

Still in 1964, Alexandre Rodrigues presented a thesis for livre docência (a title which is the equivalent of the German privatdozent) at USP, where a proof is given of the existence and uniqueness of submanifolds in Euclidean spaces, given the first and second fundamental forms and the normal connection. Thus, in this brief seven-year period, more was produced in Differential Geometry than in all of Prehistory. And the process gained momentum.

In 1966, Nathan Moreira dos Santos concluded his Doctorate at MIT, under the supervision of I. Singer, with a thesis that dealt with the conjugate locus in a Riemannian manifold. This thesis was published in the Anais da Academia Brasileira de Ciências, (39 [1967], 19-26.) The following year, A. Gervasio Colares received the Doctoral degree at Boston University, under the supervision of W. Ambrose, with a thesis entitled "On a prehilbert manifold of curves and minimal surfaces".

In 1969, Edgar Harle of USP finished his Doctorate at Berkeley under the supervision of S. Kobayashi. The thesis entitled "Rigidity of hypersurfaces of constant scalar curvature", was published in the Journal of Differential Geometry 8 (1971), 85-111.

In 1967, I went to Berkeley for a post-doctorate on a Guggenheim Fellowship. Elon Lima was also there and we worked together on a paper that was published in the Archiv der Mathematik in 1969 (20 [1969], 173-175.) A paper that I wrote afterwards on hypersurfaces

[1] Translated into English as: *S.S. Chern: Mathematical Influences and Reminiscences* in Chern - A Great Geometer of the Twentieth Century. Ed. S.T. Yau. International Press, Hong Kong, 1992

of a Hilbert space with positive curvature was published prior to that, in 1968, in the Journal of Differential Geometry (2 [1968], 355-362).

This post-doctorate period was one of the most active in my professional life. I wrote a paper on minimal surfaces in collaboration with my former advisor S.S. Chern and with S. Kobayashi, which was the starting point for my extensive interest in minimal surfaces. I also worked on a paper with N. Wallach on minimal immersions of spheres in spheres, published in the Annals of Mathematics (13 [1971], 43-62), thus beginning a line of investigation that has lasted until today. All in all, I wrote six papers at that time.

I know of no other papers on Differential Geometry written by Brazilian mathematicians outside of the ones mentioned here during the period 1957 to 1970.

A disappointing aspect during this 1957-1970 period is that all the papers published by Brazilian mathematicians on Differential Geometry, without exception, were written abroad. In order to have autonomous research, it would be necessary that high quality papers be written in Brazil. This leads us to the Research Consolidation Period, which started in 1970.

In 1970, IMPA reorganized its Graduate Program and inaugurated a Doctoral Program, which included, among other fields, Differential Geometry. My first student in 1972 was Keti Tenenblat, today an eminent mathematician. Her thesis was published in the Archiv der Mathematik in 1973 (24 [1973], 317-319.) The following year Edmilson Pontes and Rubens Leão de Andrade were given the Doctoral degree under my supervision. Edmilson's thesis was published in the Bulletin of A.M.S., 80 (1974), 581-583, and Rubens' in the Journal of Differential Geometry, 10 (1975), 491-499. In 1972, I published a paper with B. Lawson on the spherical image of convex surfaces (Proceedings of A.M.S., 31 [1972], 635-636.) All these papers had been written in Brazil and were a definite step towards doing research in Differential Geometry in this country.

During this period, various facts helped consolidate the research program. Two Brazilian mathematicians - J. Lucas Barbosa and Plínio Simões - finished their Doctorates at Berkeley with S.S. Chern (raising the number of Brazilian students of Chern to five). Lucas finished in 1972 and, after spending some time at Stanford, went back to Ceará where he helped create and develop one of the largest differential geometry groups in the country. Plínio finished in 1973 and after a while returned to USP where he began a Doctoral

Program in Differential Geometry.

Both Lucas and Plínio had completed their theses on minimal surfaces. This is an old subject in Mathematics, which had its beginnings with Lagrange in 1760 and has undergone periods of intense activity and others of relative calmness. When I was doing my post-doctorate, Chern received a manuscript of a work of J. Simons on minimal submanifolds in Riemannian manifolds. With almost prophetic vision, Chern concluded that the work would open up new perspectives in the field of minimal surfaces and gave an entire course on the subject. From then on, several students wrote their theses on the subject, other mathematicians also became interested in the topic, and minimal surfaces once again enjoyed a period of intense activity.

In 1974, IMPA hired Lucio Rodriguez, who had taken his Doctorate at Brown University with Tom Banchoff - who, in turn, had been a student of Chern and my colleague at Berkeley. This raised the number engaged in the geometry group at IMPA from one to two.

In 1975, USP hired C.C. Chen. Though he had done his Doctorate in Topology with Paul Baum at Brown University, Chen grew interested in minimal surfaces and helped consolidate the Doctorate in Differential Geometry at USP.

In 1976, the University of Campinas (UNICAMP) hired Francesco Mercuri (Franco) who had done his Doctorate in Chicago. Around the same time, Alcibiades Rigas, who had been a colleague of Franco in Chicago, was also hired by UNICAMP. This paved the way for the organization of a Doctoral Program in Differential Geometry at UNICAMP.

In this fashion, once the 1970-1983 period was over, there were full-fledged Differential Geometry Doctoral Programs at IMPA, USP and UNICAMP. Theses were in general published in internationally accredited scientific journals and these programs fulfilled the objective of producing good quality mathematics in this country. Furthermore, advisors were forced to keep themselves updated on the latest research findings, and very gradually there developed the notion that research per se was valuable as opposed to only erudition being worthwhile.

During this period IMPA produced 14 Doctors, 11 under my supervision and 3 under the supervision of Lucio Rodriguez. Only one of these remained at IMPA: Marcos Dajczer, who is today a mathematician of international repute, known for his work in isometric immersions. The rest scattered to different parts of the country where they have planted the seeds of Differential Geometry. And, as "one needs only to plant for things to grow here" [2], Differential Geometry has grown, as can be seen in the dimensions of this School of Geometry (66 conferences and communications, and 186 participants present).

The Schools of Differential Geometry have served as important stimuli for the development of research in this field. The first School was set up at IMPA in 1974. The second, in 1976, was part of a great international conference, the 3rd Latin American School of Mathematics, which included Geometry, Dynamical Systems and Topology. From then on, every two years, the School of Differential Geometry was held wherever Geometry was flourishing (Fortaleza, Campinas, São Paulo, etc.).

Also during this period, Brazilian Differential Geometry produced some research work of international repercussion, a large part of which was on minimal surfaces. Among these, I would like to point out the following (in chronological order):

1. J.L. Barbosa and M. do Carmo
   On the Size of a Stable Minimal Surfaces in R^3.
   American Journal of Mathematics, 98 (1976), 515–528.

2. M. do Carmo and C.K. Peng
   Stable Minimal surfaces in R^3 are Planes.
   Bull. of AMS, (1979), 903–906.

3. K. Tenenblat and C.L. Terng
   Backlund's theorem for n-dimensional submanifolds of R^{2n-1}.
   Annals of Math. 111 (1980), 477–490.

4. L.P.M. Jorge and F. Xavier
   A complete minimal surface in R^3 between two planes.
   Annals of Math. 112 (1980), 203–206.

5. F. Xavier
   The Gauss map of a complete non-flat minimal surface cannot ommit 7 points of

[2] From a letter of Pero Vaz de Caminha, a writer in the Portuguese fleet that discovered Brazil, to the King of Portugal

sphere.
Annals of Math. 113 (1981), 211–214.

6. C.J. Costa
Imersões mínimas completas em R^3 de gênero um e curvatura total finita.
Tese de doutorado, IMPA, 1982.
Publicação final em *Example of a complete minimal immersion in R^3 of genus one and three embedded ends.* Bol. Soc. Brasil. Mat. 15 (1984)

7. M. do Carmo and M. Dajczer
Rotation hypersurfaces in spaces of constant curvature.
Trans. Am. Math. Soc. 277 (1983), 683–709.

8. L.P.M. Jorge and W. Meeks III
The topology of complete minimal surfaces of finite total Gaussian curvature.
Topology 22 (1983),no. 2, 203–221.

For intuitive descriptions of 1, 2 and 5, see M. do Carmo, Matemática das Películas de Sabão (Mathematics of Soap Film), Ciência Hoje, Vol. 2, no. 11, 1984.

Among these papers, the one that had the greatest impact was probably that of Celso Costa's Doctoral thesis at IMPA in 1982 (he is currently a professor at the Universidade Federal Fluminense.) The story of this work is as follows:

Up until 1982, the only known examples of complete minimal surfaces, without self-intersections and with total finite curvature were the plane and the catenoid (the catenoid is the surface generated by the rotation of the catenary $y = a \cosh x$ around the axis Ox). A surface is complete if, starting from any point on a geodesic, one can mark on it any length whatsoever; intuitively, this means that the surface extends to infinity in all directions. In this case, the integral of the Gaussian curvature (which is a function defined on the surface) may not be well defined or may be infinite. We say that a surface has total finite curvature if such an integral is well defined and finite.

Various mathematicians tried, without success, to prove that only these two examples existed. In these attempts, it was seen that a third example should satisfy so many conditions as to make its existence appear impossible. Proof of this fact, however, continued to defy the best geometers of the time.

In his Doctoral thesis, under my supervision, Celso Costa wrote the equations of a candidate to a third example. The candidate satisfied all necessary conditions but one - that the surface could not have self-intersections in the finite part (which should have been the easiest to prove; that there are no intersections outside of the ball is, in general, the hardest part.) Regardless of not having obtained the complete result, what had been done was sufficiently interesting to justify a Doctoral thesis, which he presented in 1982. When I looked for him to insist on the publication of the thesis, he had disappeared. I found out later that he was in a hippie colony in Porto Seguro.

In 1984, two American mathematicians, W. Meeks and D. Hoffman, finally demonstrated that the nucleus of Celso's surface did not have self-intersections and constructed several other examples. Celso's surface, which is today called "the Costa Surface", acquired then great notoriety, made the cover of several scientific journals and is in the frontispiece of a recent treatise on minimal surfaces (U. Kierkes, S. Hildebrandt, A. Kuester, O. Wohlrab. Minimal Surfaces, Springer-Verlag, Heildeberg 1994, 2 volumes.)

To conclude, we would like to mention that Hoffman and Meeks' proof that Celso's surface is embedded, was inspired by the use of computer graphics. With such resources, they were able to transform Celso's equations into a figure and "saw" (Figure 2) that the nucleus of the surface was embedded. Computer graphic resources enabled the placing of the surface in various positions and so they were able to verify that it had many symmetries. Based on these symmetries, it was possible to develop a rigorous mathematical proof showing that the surface was, in fact, embedded. This interaction between Pure and Applied Mathematics is a frequent aspect in Mathematics which, according to the physicist Eugene Wigner "is a wonderful gift that we neither understand nor deserve"[3].

On this note, I conclude my account. I hope to have conjured up a picture, however faint, of the beginnings and the consolidation of research in Differential Geometry in Brazil. For completeness, I have included at the end of this account a list of research papers on Differential Geometry (as defined here), written by Brazilian mathematicians, from 1893

[3]From *The unreasonable effectiveness of Mathematics in the natural sciences* , by E. Wigner, Comm. on Pure and Appl. Math., 23 (1960), 1-14.

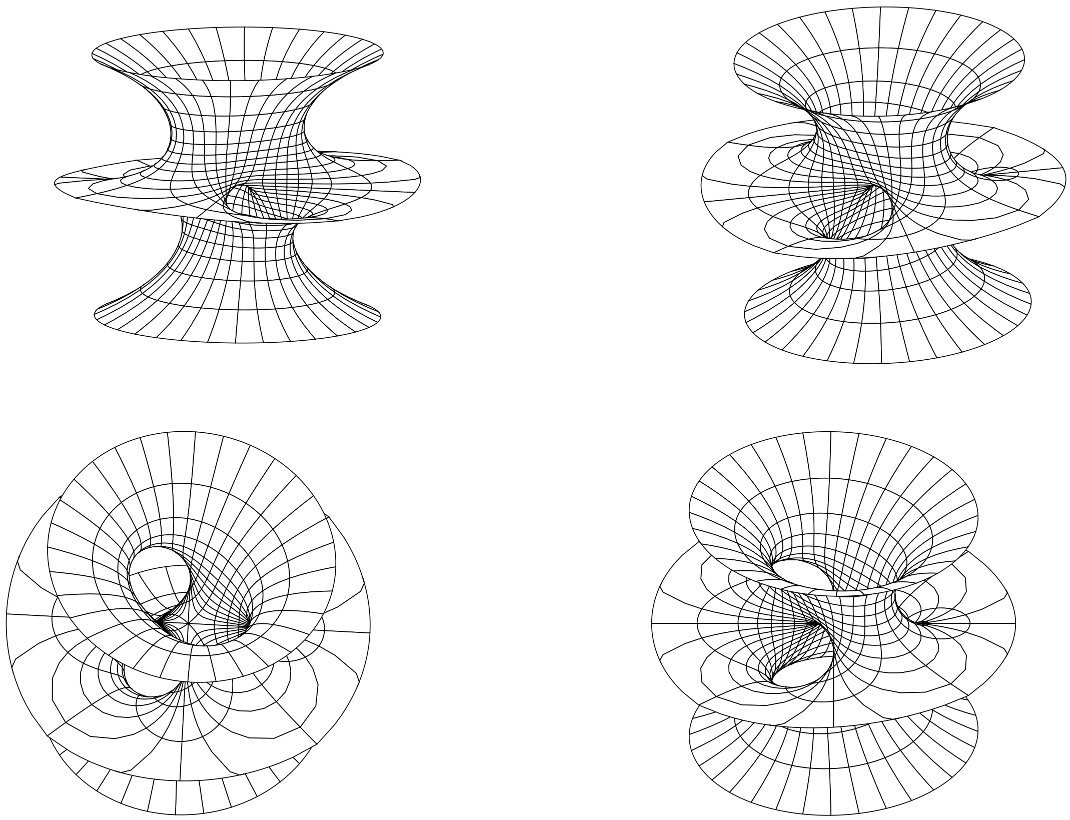

Figure 2: The Costa surface - From Fig. 20, Chapter 3, Section 3.8 of the book "Minimal Surfaces" by Dierkes et al.. Images produced by Konrad Polthier and Ulrich Reitebuch with software JavaView. JavaView is accessible at http://www-sfb288.math.tu-berlin.de/vgp/javaview/.

to 1993; it gives an idea of the rate of growth of research on Differential Geometry in this country.

I also hope that other accounts may follow this one, enlarging the panorama presented here and introducing other aspects of Geometry that the lack of time and knowledge have not permitted me to broach.

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en impossible to compile the 1883-1983 bibliography.

Research works in Differential Geometry written by Brazilian mathematicians [4] from 1883 a 1983.

1898

1. de Alencar, Otto, *A superfície mínima de Riemann de geratriz circular*, Revista da Escola Politécnica, 3 (1898), 137–144.

1929

1. Gama, Lélio, *Estudo sobre as linhas geodésicas*, Thesis to the Astronomy Chair at the Polytecnical School, 1929.

1958

1. Rodrigues, A. A. Martins, *Characteristic classes of complex homogeneous spaces.* Bol. Soc. Mat. São Paulo 10 (1955), 67–86, published in 1958 [5].

1963

1. do Carmo, Manfredo P., *The cohomology ring of certain kählerian manifolds.* An. Acad. Brasil. Ci. 35 (1963), 149–151.

1964

1. Amaral, Léo, *Hypersurfaces in non-euclidean spaces*, Ph. D. thesis, University of California, Berkeley, USA (1964).
2. Rodrigues, A. A. Martins, *Congruência de subvariedades de um espaço euclidiano*, Thesis of "Livre-Docência" presented to the Polytechnical School of USP (1964).

1965

[4] Foreign mathematicians definitely settled in Brazil are considered here as Brazilians.

[5] Here and elsewhere, this remark means that the periodical correspondiing to a certain year was actually published in the indicated date

1. do Carmo, Manfredo P., *The cohomology ring of certain kählerian manifolds.* Ann. of Math. (2), 81 (1965), 1–14.

1967

1. Colares, A. G., *On a prehilbert manifold of curves and minimal surfaces.* Ph. D. thesis, Boston University, Boston, USA (1967).

2. Moreira dos Santos, Nathan, *On the conjugate locus of a Riemannian manifold.* An. Acad. Brasil. Ci. 39 (1967), 19–26.

1968

1. do Carmo, Manfredo P., *Positively-curved hypersurfaces of a Hilbert space.* J. Differential Geometry 2 (1968), 355–362.

1969

1. do Carmo, Manfredo P.; Lima, Elon, *Isometric immersions with semi-definite second quadratic forms.* Arch. Math. (Basel) 20 (1969), 173–175.

2. do Carmo, Manfredo P.; Wallach, Nolan R., *Minimal immersions of spheres into spheres.* Proc. Nat. Acad. Sci. U.S.A. 63 (1969), 640–642.

1970

1. Chern, S. S.; do Carmo, Manfredo P.; Kobayashi, S., *Minimal submanifolds of a sphere with second fundamental form of constant length.* 1970 Functional Analysis and Related Fields (Proc. Conf. for M. Stone, Univ. Chicago, Chicago, Ill., 1968) Springer, New York, 59–75.

2. do Carmo, Manfredo P.; Wallach, Nolan R., *Minimal immersions of sphere bundles over spheres.* An. Acad. Brasil. Ci. 42 (1970), 5–9.

3. do Carmo, Manfredo P.; Warner, F. W., *Rigidity and convexity of hypersurfaces in spheres.* J. Differential Geometry 4 (1970), 133–144.

4. do Carmo, Manfredo P.; Wallach, Nolan R., *Representations of compact groups and minimal immersions into spheres.* J. Differential Geometry 4 (1970), 91–104.

1971

1. do Carmo, Manfredo P.; Lima, E., *Immersions of manifolds with non-negative sectional curvatures.* Bol. Soc. Brasil. Mat. 2 (1971), no. 2, 9–22.

2. do Carmo, Manfredo P.; Wallach, Nolan R., *Minimal immersions of spheres into spheres.* Ann. of Math. (2) 93 (1971), 43–62.

3. Harle, Carlos Edgard, *Rigidity of hypersurfaces of constant scalar curvature.* J. Differential Geometry 5 (1971), 85–111.

4. Tenenblat, Keti, *On isometric immersions of Riemannian manifolds.*Bol. Soc. Brasil. Mat. 2 (1971), no. 2, 23–36.

1972

1. do Carmo, Manfredo P.; Lawson, B., *Spherical images of convex surfaces.* Proc. Amer. Math. Soc. 31 (1972), 635–636.

2. Nomizu, Katsumi; Rodriguez, Lucio, *Umbilical submanifolds and Morse functions.* Nagoya Math. J. 48 (1972), 197–201.

1973

1. Barbosa, J.L., *On minimal immersions of S^2 in S^{2m}.* Bull. Amer. Math. Soc. 79 (1973), 110–114.

2. Chen, Chi Cheng, *On the residues of meromorphic vector-fields.* Notas e Comunicações de Matemática, No. 51. Universidade Federal de Pernambuco, Instituto de Matemática, Recife, (1973).

3. Mendes, Roberto M.N., *Symmetries of spherical harmonics* Ph. D. thesis, University of California, San Diego, USA (1973).

4. Mercuri, Francesco, *Odd dimensional manifolds with regular conjugate locus.* Proc. Amer. Math. Soc. 38 (1973), 441–442.

5. Moreira dos Santos, Nathan, *The local structure of the conjugate locus*, Soc. Bras. Mat., Atas da Reunião de Geometria e Topologia - UNICAMP, Coleção Atas vol. 6 (1973), 137-155.

6. Tenenblat, Keti, *An estimate for the length of closed geodesics on a Riemannian manifold.* Arch. Math. (Basel) 24 (1973), 317–319.

7. Tenenblat, Keti, *On Klingenberg's theorem.* Bol. Soc. Brasil. Mat. 4(1973), no. 2, 139–146.

8. Tenenblat, Keti, *On the Rauch comparison theorem for volumes.* Bol. Soc. Brasil. Mat. 4 (1973), no. 1, 31–39.

1974

1. Barbosa, J. L.; do Carmo, Manfredo P., *Stable minimal surfaces.* Bull. Amer. Math. Soc. 80 (1974), 581–583.

2. Pontes, Edmilson, *Isometric minimal immersions of* $S^3(a)$ *in* $S^N(1)$. Bull. Amer. Math. Soc. 80 (1974), 1239–1242.

3. Simões, Plínio, *On a class of minimal cones in* R^n. Bull. Amer. Math. Soc. 80 (1974), 488–489.

1975

1. de Andrade, Rubens Leão, *Complete convex hypersurfaces of a Hilbert space.* J. Differential Geometry 10 (1975), no. 4, 491–499.

2. Barbosa, J. L., *On minimal immersions of* S^2 *into* S^{2m}. Trans. Amer. Math. Soc. 210 (1975), 75–106.

3. do Carmo, Manfredo P.; Nowosad, P., *Stability of minimal hypersurfaces.* An. Acad. Brasil. Ci. 47 (1975), no. 1, 27.

4. Mercuri, Francesco; Palmieri, Giuliana, *Problems in extending Morse theory to Banach spaces.* Boll. Un. Mat. Ital. (4) 12 (1975), no. 3, 397–401.

1976

1. Barbosa, J. L.; do Carmo, Manfredo P., *On the size of a stable minimal surface in* R^3. Amer. J. Math. 98 (1976), no. 2, 515–528.

2. Mercuri, Francesco, *Closed geodesics on Finsler manifolds.* Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 60 (1976), no. 2, 111–118.

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