# SOME REMARKS ON CYLINDRICALLY BOUNDED $H$-SURFACES WITH COMPACT BOUNDARY

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The motivation for this paper was the following question: does a properly embedded $H$-surface $M$, with $\partial M$ compact, have finite topology if it is cylindrically bounded? We do not know the answer to this question. The results we obtain here add some understanding to the problem. In some sense (theorems 1.1, 1.2) if such an $M$ has infinite topology then a stack of spheres forms as a limit and one is led to the problem of studying handles that are converging to the point of contact of two tangent spheres. If $M$ has bounded curvature these handles do not exist so $M$ has finite topology (theorem 2.1). We thank Antonio Ros for interesting conversations.

When $M$ is an immersed small compact $H$-surface whose boundary projects onto a convex planar curve $\Gamma$, then we prove a Rado-type theorem: $M$ is an embedded disk; a graph over the planar domain with boundary $\Gamma$.

Most of our results extend to surfaces in $\mathbb{H}^3$, when $H > 1$. We indicate the modifications in the Euclidean proofs that work in hyperbolic space. We assume $H > 0$ throughout this paper.

## 1. Cylindrically Bounded Surfaces in $\mathbb{R}^3$ and $\mathbb{H}^3$

Let $\mathbb{R}^3_+ = \{x_3 \geq 0\}$ and $P = \{x_3 = 0\}$; we note $\sigma = (0,0,0)$ and $\gamma$ denotes the vertical line through $\sigma$. Let $D(r) = \{x_1^2 + x_2^2 < r^2, x_3 = 0\}$.

**Theorem 1.1**. *Let $M_n \subset \mathbb{R}^3_+$ be noncompact complete properly embedded $H$-surfaces, and $\partial M_n \subset D(r_n)$ with $r_n$ a sequence converging to zero. If the $M_n$ are contained in a vertical cylinder of $\mathbb{R}^3_+$, outside of some compact set, then there* *is a subsequence of* $M_n$ *that converges to the stack of spheres of mean curvature* $H$*, tangent to* $P$ *at* $\sigma$*; the convergence is uniform on compact subsets of* $\mathbb{R}^3_+\backslash\gamma$.

**Proof**: Let $Z$ be a solid vertical cylinder in $\mathbb{R}^3_+$ with axis $\gamma$. One can choose $Z$ large enough such that it contains the $M_n$. We will construct compact domains $\Omega_m \subset Z$ such that the complement of $\bigcup_m^\infty \Omega_m$ in $Z$ is $\gamma$ and such that $\Omega_m \subset \Omega_{m+1}$. The idea is then to show that for each $m$ and $n$ sufficiently large, the part of $M_n$ in $\Omega_m$ is a graph (we make this precise below). So one has uniform area and curvature estimates for this part of $M_n$. Then standard compactness techniques yield a subsequence of $M_n$ that converges on compact sets of $\mathbb{R}^3_+\backslash\gamma$.

Let $\varepsilon$ and $r$ be positive reals and let $C(\varepsilon, r)$ be the part in $\mathbb{R}^3_+$ of the solid vertical cone with vertex on $\sigma \times (-\infty, 0]$ such that $C(\varepsilon, r) \cap P = D(r)$ and the outer angle between $\partial C(\varepsilon, r)$ and $P$ along $\partial D(r)$ is $\frac{\pi}{2} - \varepsilon$. The intersection of $Z$ and the complement of $C(\varepsilon, r)$ in $\mathbb{R}^3_+$ is a bounded domain since $C(\varepsilon, r)$ and $Z$ have the same axis $\gamma$. Then we define $\Omega_m$ to be the closure of the set $\left\{\mathbb{R}^3_+\backslash C\left(\varepsilon = \frac{1}{m},\ r = \frac{1}{m}\right)\right\} \cap Z$. Notice that $\bigcup_m^\infty \Omega_m = Z\backslash\gamma$.

Now for $m$ fixed and $n$ large $\partial M_n \subset D(r)$. Let $\beta(t)$, $0 \leq t < \infty$, be a geodesic in $P$ starting at $\sigma$, parametrized by arc length. Let $Q(t)$ be a family of vertical planes such that $Q(t)$ intersects $\beta$ orthogonally at $\beta(t)$. Let $T(t)$ be a plane that makes an angle $\varepsilon$ with $Q(t)$ at $\beta(t)$ and $T(t) \cap P = Q(t) \cap P$. We tilt $Q(t)$ to $T(t)$ so that $T(t) \cap \mathbb{R}^3_+ \cap \gamma$ is empty (there are two ways to tilt $Q(t)$). Clearly for $t$ large, $T(t)$ is disjoint from $M_n$. Apply the Alexandrov reflection process to $M_n$ and the planes $T(t)$ (cf. [RR]). One can translate $T(t)$ until $\partial D(r)$ and the part of $M_n$ swept out by $T(r)$ is a graph over $T(r)$. So one has uniform area and curvature estimates (cf. [RR]) for this part of $M_n$ (the area of $T(r) \cap \Omega_m$ is bounded by the geometry of $\Omega_m$). Now standard compactness techniques yield a subsequence (which we also call) $M_n$ that converges on compact subsets of $\Omega_m$. We can do this for each $m$ and by a diagonalisation process we then find a subsequence $M_n$ that converges on compact subsets of $\mathbb{R}^3_+\backslash\gamma$. The limit is either empty or a surface $M$ of mean curvature $H$.

Now we will prove that the limit is not empty. Suppose, on the contrary, that the limit is empty, then for $n$ large $M_n$ would be uniformly close to $\gamma$ on compact sets. Therefore $M_n$ stays inside of the vertical cylinder of radius $\frac{1}{2H}$ with axis $\gamma$.

The idea is to deform a compact annulus $A$ of vertical length $\frac{\pi}{H}$ of this cylinder in the set of Delaunay surfaces of mean curvature $H$. The vertical translation periods of the Delaunay surfaces start at $\frac{\pi}{H}$ (for the cylinder) and decrease to $\frac{2}{H}$ (for the stack of spheres).

On each Delaunay surface, choose continuously a compact annulus $A(t)$, with boundary two horizontal circles of vertical distance the period of the Delaunay surface. Also choose $A(t)$ so that the $\partial A(t)$ are circles where the Delaunay surface has maximum width. So $A(0) = A$ and $A(t)$ converges to the part $B$ of a stack of spheres bounded by two successive horizontal circles of radius $\frac{1}{H}$ ·

Since $\partial A(t)$ is always outside $C$ for $t > 0$, and $A(t)$ pinches in to a point on the vertical axis $\gamma$ as $A(t)$ converges to $B$, there will be a first point of contact of some $A(t)$ with $M_n$. Then the maximum principle implies that both surfaces coincide, which is impossible.

Therefore we can assume the $M_n$ converge to $M$. For each $\varepsilon > 0$, the planes $T(t)$ can be moved up to $\partial D(\varepsilon)$ and the symmetries of $M$ by these planes do not touch $M$ (since this holds for $M_n$, $n$ large). So this works up till $\varepsilon = 0$ by continuity and $M$ is a rotational surface about $\gamma$; a connected Delaunay surface. $M$ is not compact, thus $M$ is not a sphere. The point $\sigma$ is on $M$ so $M$ is a stack of spheres of radius $\frac{1}{H}$ passing through $\sigma$.

$\square$

**Remark 1**: This theorem still holds if one assumes $\partial M_n \subset B(r_n)$, $B(r_n)$ Euclidean 3-balls centered at $\sigma$, with $r_n \to 0$. Also one must assume $M_n \subset \mathbb{R}^3_+$. The proof is the same.

Observe that we can interpret this theorem as follows. Let $M_n \subset \mathbb{R}^3_+$ be a sequence of $H_n$-surfaces with $H_n \to 0$. Assume each $M_n$ is vertically cylindrically bounded (not necessarily the same cylinder for each $M_n$) and assume that there is some compact $B$ with $\partial M_n \subset B$ for each $n$. Then fix $\sigma \in B$ and do a homothety of $M_n$ at $\sigma$ to obtain $\widehat{M}_n$ of $\widehat{H}_n = 1$. Then $\widehat{M}_n$ converge to the stack of spheres.

**Theorem 1.2**: *Let* $C_1 = \{x_1^2 + x_2^2 = 1, x_3 = 0\}$, *and* $C_2 = C_1 + (0,0,2)$. *Let* $M_n$ *be a sequence of embedded compact* $H = 1$ *surfaces,* $M_n \subset \{0 \leq x_2 \leq 2\}$, $\partial M_n = \Gamma_1^n \cup \Gamma_2^n$, *with* $\Gamma_1^n$ *converging uniformally to* $C_1$ *in* $\{x_3 = 0\}$, *and* $\Gamma_2^n$ *converging uniformally to* $C_2$ *in* $\{x_3 = 2\}$. *Then* $M_n$ *converges to the union of the two hemispheres of radius one, with boundary* $C_1 \cup C_2$ *and passing through the (singular) point* $(0,0,1)$.

**Proof**: We will apply the Alexandrov reflection technique to $M_n$ with vertical planes.

Let $\varepsilon > 0$ and let $P(\varepsilon)$ be the vertical plane $\{x_1 = \varepsilon\}$. Since $\Gamma_1^n$ is uniformally close to $C_1$, the tangent vector along $\Gamma_1^n \cap \{x_1 \geq \varepsilon\}$ is uniformally bounded away from $\vec{v} = (1,0,0)$ for $n$ large. So, by reflection of $\Gamma_1^n \cap \{x_1 \geq \varepsilon\}$ with respect to $P(\varepsilon)$, the tangent vector of the image will be bounded away from $-\vec{v}$. For the same $\varepsilon$ and $n$ large enough, the above is also true for $\Gamma_2^n \cap \{x_1 \geq \varepsilon\}$.

Now consider Alexandrov reflection with vertical planes $P(t) = \{x_1 = t\}$ for $t \geq \varepsilon$. For $t$ large, $P(t)$ is disjoint from $M_n$.

As the planes $P(t)$ approach $P(\varepsilon)$, consider the first possible point of contact of $M_n$ with its symmetry through $P(t)$. Since $M_n$ is in the slab $\{0 \leq x_3 \leq 2\}$, the first contact can not arise from the symmetry of an interior point of $M_n$ touching a point of $\partial M_n$. So the first possible contact would be the symmetry of an interior point touching an interior point. This would give a vertical plane of symmetry $P(t)$ for $t \geq \varepsilon$, which is impossible since $\partial M_n$ is never orthogonal to $P(t)$ for $t \geq \varepsilon$.

Another possibility is that the image of a boundary point touches the boundary. However, for $n$ large, $\Gamma_i^n$ is uniformally close to the circle $C_i$ and so the symmetry through $P(t)$ of the short arc $\Gamma_i^n \cap \{x_1 > t\}$ cannot meet $\Gamma_i^n \cap \{x_1 < t\}$. Therefore, there is no point of contact until $t = \varepsilon$ and the part of $M_n$ swept out by the planes $P(t)$, $t \geq \varepsilon$, is a graph over $P(\varepsilon)$.

This is true for all $\varepsilon > 0$ and $n$ large enough. Thus one has uniform curvature and area estimates for this part of $M_n$.

We can repeat this reasoning for each direction $(\cos\sigma)x_1 + (\sin\sigma)x_2$, $0 < \sigma \leq 2\pi$, hence by standard compactness techniques we obtain a subsequence of $M_n$ converging on all compact sets of $\mathbb{R}^3 \backslash (x_3 - \text{axis})$. The limit is either empty or a compact surface $M$ of mean curvature 1. To see that the limit is not empty, we use the same argument as in the proof of theorem 1.1. (Now we work with a family of Delaunay surfaces of mean curvature 2, so the height of one period is between 1 and $\frac{\pi}{2} < 2$). So the limit exists.

As we have shown before, for each $\varepsilon > 0$ and for each family of vertical planes, the symmetries of $M_n$ with respect to $P(\varepsilon)$, $n$ large enough, do not touch $M_n$. So this holds for $M$.

By continuity, this works up till $\varepsilon = 0$ and so $M$ is a rotational surface (hence part of a Delaunay surface) about the $x_3$-axis which can have self-intersections at most on the $x_3$-axis. $M$ is compact, of height 2, and the boundary of $M$ are the two circles $C_1 \cup C_2$. This completes the proof.

$\square$

## 1.2. The Hyperbolic Case

Let $\sigma$ be a point in the hyperbolic plane $P$ and let $\gamma(t)$, $-\infty \leq t \leq \infty$, be the geodesic orthogonal to $P$ at $\sigma$. Parametrize $\gamma$ such that $\gamma(0) = \sigma$ and $\gamma(t) \subset \mathbb{H}^3_+$ = a half-space determined by $P$, for $t \geq 0$. $t$ will be positive. Note by $Z$ a solid Killing cylinder in $\mathbb{H}^3$ with axis $\gamma$, i.e. the integral curves of the Killing vector field associated with the hyperbolic translation along the geodesic $\gamma$ at a constant distance from $\gamma$.

**Remark 2**: The hypothesis that $M$ is not compact and contained in a solid half-cylinder $Z$, implies that $M$ has mean curvature bigger than 1 [S].

**Theorem 1.3**: *Let $M_n \subset \mathbb{H}^3_+$ be noncompact complete properly embedded $H$-surfaces, and $\partial M_n \subset D(r_n) \subset P$ where $D(r_n)$ are disks of hyperbolic radius $r_n$ centered at $\sigma$ with $r_n$ a sequence converging to zero. If the $M_n$ are contained in*

*$Z \cap \mathbb{H}^3_+$, outside of some compact set, then there is a subsequence of $M_n$ that converges to the stack of spheres of mean curvature $H$, tangent to $P$ at $\sigma$; the convergence is smooth on compact subsets of $\mathbb{H}^3_+ \backslash \gamma$.*

**Remark 3**: The proof of Theorem 1.3 is similar to the corresponding proof of Theorem 1.1 in Euclidean space apart from the fact that it works without using tilted planes. The only arguments not obvious, are the choices of the domains $\Omega_m$ and the planes $P(t)$.

**Proof**: One chooses $Z$ large enough such that it contains the $M_n$. We construct compact domains $\Omega_m \subset Z$ such that the complement of $\bigcup_m^\infty \Omega_m$ in $Z$ is $\gamma$ and such that $\Omega_m \subset \Omega_{m+1}$. The idea is then to show that for each $m$ and $n$ sufficiently large, the part of $M_n$ in $\Omega_m$ is a geodesic graph. Then one has uniform area and curvature estimates for this part of $M_n$. Now standard compactness techniques yield a subsequence of $M_n$ that converges on compact sets of $\mathbb{H}^3_+ \backslash \gamma$.

Let $r$ be a positive real and let $C(r)$ be the part in $\mathbb{H}^3_+$ of the solid geodesic cylinder over the disk in $P$ of radius $r$ centered at $\sigma$; this means

$$C(r) = \bigcup_{p \in D(r)} \gamma_p \cap \mathbb{H}^3_+$$

where $\gamma_p$ is the unique geodesic through $p$ orthogonal to $P$. The intersection of $Z$ and the complement of $C(r)$ in $\mathbb{H}^3_+$ is a bounded domain since the point at infinity of $Z$ is contained in the set of points at infinity of $C(r)$. Thus we define $\Omega_m$ to be the closure of the set

$$\{\mathbb{H}^3_+ \backslash C(r = \text{ arcoth } m)\} \cap Z.$$

Notice that as $m$ goes to $\infty$, $\bigcup_m^\infty \Omega_m = Z \backslash \gamma$.

Now for $m$ fixed and $n$ large $\partial M_n \subset D(r)$. Let $\beta(t)$, $0 \leq t < \infty$, be a geodesic in $P$ starting at $\sigma$. Let $Q(t)$ be a family of planes such that $Q(t)$ intersects $\beta$ orthogonally at $\beta(t)$. Clearly for $t$ large, $Q(t)$ is disjoint from $M_n$. Apply the Alexandrov reflection process to $M_n$ and the planes $Q(t)$ (cf. [NS]). One can translate $Q(t)$ along $\beta$ until $\partial D(r)$ and the part of $M_n$ swept out by $Q(r)$ is a geodesic graph over $Q(r)$. So one has uniform area and curvature estimates (cf. [NS]) for this part of $M_n$ (the area of $Q(r) \cap \Omega_m$ is bounded by the geometry of $\Omega_m$). Now standard compactness techniques yield a subsequence (which we also call) $M_n$ that converges on compact subsets of $\Omega_m$. We can do this for each $m$ and by a diagonalisation process we find a subsequence $M_n$ that converges on compact subsets of $\mathbb{H}^3_+ \setminus \gamma$. The limit is either empty or a surface $M$ of mean curvature $H$.

Now we will prove that the limit is not empty. Suppose, on the contrary, that the limit is empty, then for $n$ large $M_n$ would be uniformly close to $\gamma$ on compact sets. Therefore $M_n$ stays inside of the Killing cylinder of radius $\frac{1}{2}$ arcoth $H$ with axis $\gamma$.

As in the proof of theorem 1.1 deform a compact annulus, say $R$, of height 2arcoth $H$, of this cylinder along the one-parameter family of Delaunay surfaces of constant mean curvature $H$. The family converges to one period of a chain of spheres, so there must be a Delaunay surface in the family that first makes one-sided tangential contact at an interior point of $M_n$. Then the maximum principle implies that both surfaces coincide, which is a contradiction.

Therefore we can assume the $M_n$ converge to $M$. For each $r > 0$, the planes $Q(t)$ can be moved up to $\partial D(r)$ and symmetries of $M$ by these planes do not touch $M$ (since this holds for $M_n$, $n$ large). So this works up till $r = 0$ by continuity and $M$ is a rotational surface about $\gamma$. $M$ is not compact, thus $M$ is not a sphere. $\partial M$ is a single point so $M$ is the limit of the Delaunay surfaces; this means $M$ is a stack of spheres of radius arcoth $H$ passing through $\sigma$.

□

Theorem 1.3 remains true, if one assumes that the $\partial M_n$ are contained in a horosphere. More precisely, let L be a horosphere in $\mathbb{H}^3$ and let $\mathfrak{L}$ be the noncompact component of $\mathbb{H}^3$ bounded by L such that the mean curvature vector of L points towards $\mathfrak{L}$. Let $\sigma$ be a point in L and let $\gamma$ be the geodesic orthogonal to L at $\sigma$. Denote by $Z$ a solid Killing cylinder in $\mathbb{H}^3$ with axis $\gamma$.

**Theorem 1.4**. *Let $M_n \subset \mathfrak{L}$ be noncompact complete properly embedded $H$-*

*surfaces, and $\partial M_n \subset D(r_n) \subset L$ where $\partial D(r_n)$ are hyperbolic circles of radius $r_n$ centered on $\gamma$ with $r_n$ a sequence converging to zero. If the $M_n$ are contained in $Z \cap \mathfrak{L}$, outside of some compact set, then there is a subsequence of $M_n$ that converges to the stack of spheres of mean curvature $H$, tangent to $L$ at $\sigma$; the convergence is uniform on compact subsets of $\mathfrak{L}\backslash\gamma$.*

The proof of this is the same as before and we only sketch how to construct the domain $\Omega_m$. For $r > 0$, the domain $D(r)$ in Ł determines a plane, noted $P_r$, such that $Ł \cap P_r = \partial D(r)$. Notice that if $r$ goes to zero, $P_r$ converges to the plane of $\mathbb{H}^3\backslash\mathfrak{L}$ tangent to Ł at $\sigma$. Let $C(r)$ be the geodesic solid cylinder over the disk in $P_r$ bounded by $\partial D(r)$ and orthogonal to $P_r$. Then we define $\Omega_m$ by $\Omega_m = \{\mathfrak{L}\backslash\mathfrak{C}(\mathfrak{r})\} \cap \mathfrak{Z}$ where $r = \operatorname{arcoth}(m)$. For each $m$, the family of planes $Q(t)$ (cf. proof of Theorem 1.3) is now defined with respect to $P_{r=\operatorname{arcoth} m}$.

## 2. Cylindrically Bounded with Bounded Curvature

**Theorem 2.1**. *Let $M$ be a properly embedded $H$-surface with compact boundary. If $M$ is cylindrically bounded and $M$ has bounded curvature then $M$ has finite topology.*

**Proof**. Let $Z$ be a vertical cylinder which contains $M$. Suppose on the contrary that the topology of $M$ is not finite. Then there is a sequence of points $p_n \in M$ such that the height function $x_3$ has a critical point at each $p_n$ of negative index. $M$ is properly embedded so we can assume $x_3(p_n) \to +\infty$.

Now do a vertical translation of $M$ to the surface $M_n$ placing $p_n$ at $q_n$ on the plane $x_3 = 0$. The family $M_n$ has uniform local area bounds since any $H$-surface in a cylinder has linear area growth ([KKS]). We are assuming bounded curvature, so a subsequence of $M_n$ converges to a (non empty) $H$-surface $M_\infty$. Notice that $M_\infty$ has no boundary so it is a Delaunay surface ([KKS]).

Let $q_\infty$ be a limit point of $q_n$. Each $M_n$ has a horizontal tangent plane at $q_n$, so $M_\infty$ also is horizontal at $q_\infty$. Thus $M_\infty$ is a stack of spheres and $q_\infty$ is a singular point.

Now each $M_n$ has strictly negative curvature in a neighborhood of $q_n$ and $M_n$ converges uniformally on compact sets to $M_\infty$ , away from the singularities so this is impossible.

**Remark 4**: This last theorem is valid in $\mathbb{H}^3$; the proof is the same.

## 3. A Rado Type Theorem

We shall consider immersed compact $H$-surfaces $M$ in $\mathbb{R}^3$ with $\partial M = \Gamma$ a Jordan curve lying on a vertical cylinder $Z$ of $\mathbb{R}^3$. Assume $\Gamma$ has a one-to-one projection onto a planar convex curve $\Gamma_0$ in a horizontal plane. We say that $M$ is a small $H$-surface if $M$ is contained in some $B(p,r)$ (the closed Euclidean ball in $\mathbb{R}^3$ centered at $p$ of radius $r$) for some $p \in \mathbb{R}^3$ and $r < \frac{1}{H} \cdot$

Little is known about the geometry and the topology of such $M$ in terms of that of $\Gamma$. In 1932, Rado treated the case of immersed compact minimal surfaces with boundary a curve $\Gamma$ as above. He proved that such a surface is a vertical graph and therefore an embedded disk. In the same spirit we will show that, for $H$ sufficiently small, a small $H$-surface $M$ is a graph over the planar domain $\Omega$ bounded by $\Gamma_0$ .

**Theorem 3.1**. *Let $\Gamma$ be as defined above. There exists an $h(\Gamma) > 0$, depending only on $\Gamma$, such that whenever $M \subset \mathbb{R}^3$ is a small $H$-surface bounded by $\Gamma$, with $0 < H < h(\Gamma)$, then $M$ is a graph; in particular, $M$ is an embedded disk.*

**Proof**. It follows from the maximum principle that, for small $H$-surfaces $M$, one has $M \subset \bigcap_\alpha B_\alpha$ , where $B_\alpha$ denotes the family of balls $B(q,\rho)$, $q \in \mathbb{R}^3$, $\rho \leq \frac{1}{H}$ and $\partial M \subset B(q,\rho)$ (cf. [RR]). Let $\rho_0(q)$ be the smallest radius such that $B(q,\rho_0(q))$ contains $\Gamma$ and take the minimum $\rho_0$ over all $q \in \mathbb{R}^3$.

Let $k$ be the smallest value of the curvature of $\Gamma_0$ . Now we consider the solid vertical cylinder $C$ of radius $R$, where $\frac{1}{R} = \min\left(k, \frac{1}{\rho_0}\right)$, in a position such that $B(\rho_0) \subset C$. If $H$ is smaller than $\frac{1}{2R}$ the mean curvature of $C$ is bigger than that of $M$.

Now, by moving $\partial C$ towards $M$, we will prove that $M$ lies in $Z$. By the maximum principle, as $\partial C$ approaches $Z$ by horizontal translations, the first contact with $M$ cannot be at an interior point of $M$. Therefore no accident will occur before reaching $\Gamma$. Notice that by our choice of $R$ we are able to touch any point of $\Gamma$. Therefore $M$ is included in $Z$.

In addition, the variational techniques (chapter 15 in [GT]) applied to the constant mean curvature equation yield the existence of $H$-graphs $G$ over $\Omega$, unique with respect to the upper mean curvature vector, with boundary $\Gamma$ for $H$ smaller than a number $h_1(\Gamma) > 0$.

Let $h$ be the minimum of $\frac{1}{2R}$ and $h_1(\Gamma)$. So, for $H$ smaller than $h$, we use $G$ and $G^-$, the $H$-graph solution with respect to the lower mean curvature vector, where $G \cap G^- = \Gamma$ by the maximum principle, to conclude that $M$ must lie in the domain in $Z$ bounded by $G \cup G^-$.

Furthermore, a uniqueness result proved in ([BS]), using a flux argument, implies that $M$ must coincide with $G$ or $G^-$.

**Remark 5**: An interesting question is to study the same problem for big $H$-immersions.

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