uniformly in ν. Now we can estimate v_t^ν directly from (3.1) and obtain that

$$v_t^\nu \text{ is bounded in } L^2(0,T,H^{-2}(0,1)). \tag{3.6}$$

Taking into account compactness arguments and embedding theorems, we can see that v^ν converges strongly in $L^2(Q)$. Therefore, there exists a subsequence of $\{v^\nu\}$ which converges a.e. in Q. Then $v^\nu v_x^\nu$ converges to vv_x in the sense of distributions in Q. From (3.5) and (3.6), we conclude that

$$Lv = v_t + \frac{1}{\gamma(t)} v v_y - \frac{(y\gamma'(t) + \alpha_1'(t)}{\gamma(t)} v_y + \frac{\beta}{\gamma^2(t)} v_{yy} +$$

$$\frac{\eta}{\gamma^3(t)} v_{yyy} + \frac{\delta}{\gamma^4(t)} v_{yyyy} = \tilde{f}, \quad \text{in } L^2(0,T;H^{-2}(0,1)) \tag{3.7}$$

$$v(y,0) = v_0(y), \quad y \in (0,1). \tag{3.8}$$

Proof of uniqueness. Let v_1, v_2 be two solutions of (3.7)$-$(3.8) corresponding to the same initial data v_0, and $z = v_1 - v_2$.

Obviously,

$$z \in L^\infty(0,T;L^2(0,1)) \cap L^2(0,T;H^2(0,1)),$$

$$z_t \in L^2(0,T;H^{-2}(0,1))$$

and

$$\int_0^t (z_\tau, w)(\tau) d\tau + \int_0^t \frac{1}{\gamma(\tau)} \left([v_1 v_{1y} - v_2 v_{2y}], w\right)(\tau) d\tau$$

$$- \int_0^t \left([\frac{(y\gamma'(\tau) + \alpha_1'(\tau)}{\gamma(\tau)} z_y - \frac{\beta}{\gamma^2(\tau)} z_{yy}], w \right)(\tau) d\tau - \int_0^t \frac{\eta}{\gamma^3(\tau)} (z_{yy}, w_y)(\tau) d\tau$$

$$+ \int_0^t \frac{\delta}{\gamma^4(\tau)} (z_{yy}, w_{yy})(\tau) d\tau = 0,$$

where w is an arbitrary function from $L^2(0,T;W_2(0,1))$. Replacing w by z, we obtain the equality,

$$|z(t)|^2 + \int_0^t ([v_1^2 - v_2^2]_y, z)(\tau) d\tau + \int_0^t (\frac{\gamma'(\tau)}{\gamma(\tau)} |z(\tau)|^2 d\tau - 2 \int_0^t \frac{\beta}{\gamma^2(\tau)} |z_y(\tau)|^2 d\tau$$

$$-2 \int_0^t \frac{\eta}{\gamma^3(\tau)} (z_{yy}, z_y)(\tau) d\tau + 2 \int_0^t \frac{\delta}{\gamma^4(\tau)} |z_{yy}(\tau)|^2 d\tau = 0. \tag{3.9}$$