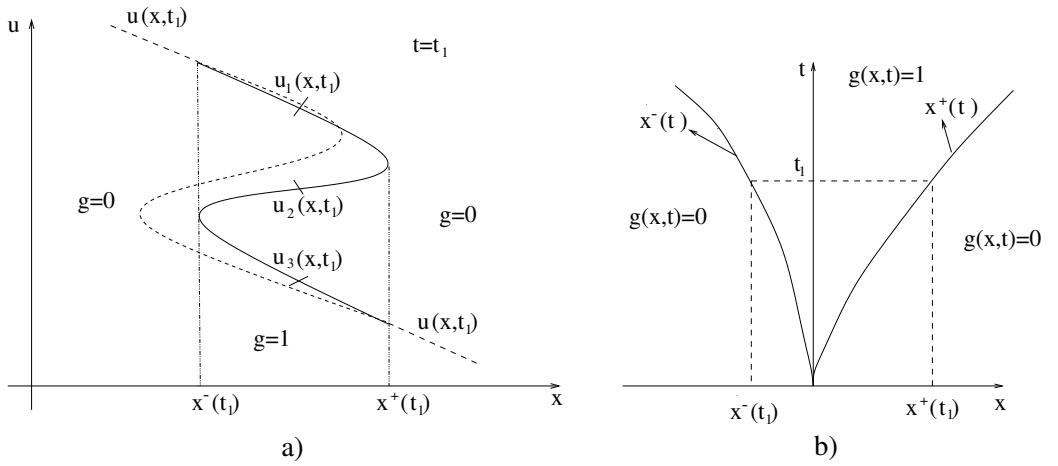

Figure 1.2: On picture a) the dashed line represents the formal solution of the Burgers equation, the continuous line represents the solution of the Whitham equations. The solution $(u_1(x,t)\,,\, u_2(x,t), u_3(x,t))$ of the Whitham equations and the position of the boundaries $x^-(t)$, $x^+(t)$ of the oscillatory zone are to be determined from the condition $u(x^-(t),t) = u_1(x^-(t),t)$, $u_x(x^-(t),t) = u_{1x}(x^-(t),t)$, $u(x^+(t),t) = u_3(x^+(t),t)$, $u_x(x^+(t),t) = u_{3x}(x^+(t),t)$ where $u(x,t)$ is the solution of the Burgers equation. Picture b) represents the functions $x^-(t)$ and $x^+(t)$ on the $x-t$ plane.

of $u_1 > u_2 > u_3$ by the relations

$$a = u_2 - u_3 \qquad s = \frac{u_2 - u_3}{u_1 - u_3}\,, \qquad \gamma = u_2 + u_3 - u_1\,. \tag{1.3}$$

The Whitham equations for the u_i, $i = 1,2,3$ read [21]

$$\partial_t u_i(x,t) + \lambda_i(u_1,u_2,u_3)\partial_x u_i(x,t) = 0, \quad i = 1,2,3\,, \tag{1.4}$$

where

$$\lambda_i(u_1,u_2,u_3) = 2(u_1+u_2+u_3) + \frac{\prod_{j\neq i,j=1}^{3}(u_i - u_j)}{u_i + \alpha_0}, \tag{1.5}$$

$$\alpha_0 = -u_1 + (u_1 - u_3)\frac{E(s)}{K(s)}$$

and $E(s)$ is the complete elliptic integral of the second kind.

The variable u_2 can vary from u_3 to u_1. The oscillatory region is bounded on one side by the point $x^-(t)$ where $u_2(x,t) = u_3(x,t)$, and on the other side by