ON THE INFINITE RADICAL OF A MODULE CATEGORY OVER A TILTED ALGEBRA

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Abstract

For an algebra A, denote by $\mathrm{rad}^\infty(\mathrm{mod}A)$ the intersection of all powers $\mathrm{rad}^i(\mathrm{mod}A)$, $i \geq 1$, of $\mathrm{rad}(\mathrm{mod}A)$. We discuss here the tilted algebras such that $\mathrm{rad}^\infty(\mathrm{mod}A)$ are nilpotent.

Resumo

Para uma álgebra A, denotemos por $\mathrm{rad}^\infty(\mathrm{mod}A)$ a intersecção de todas as potências $\mathrm{rad}^i(\mathrm{mod}A)$, com $i \geq 1$, de $\mathrm{rad}(\mathrm{mod}A)$. Discutimos aqui as álgebras tilted tais que $\mathrm{rad}^\infty(\mathrm{mod}A)$ seja nilpotente.

Let A be finite dimensional algebra over an algebraically closed field k and let $\mathrm{mod}A$ denote the category of finitely generated right A-modules. Denote by $\mathrm{rad}(\mathrm{mod}A)$ the Jacobson radical of $\mathrm{mod}A$, that is, the ideal in $\mathrm{mod}A$ generated by all non-invertible morphisms between indecomposable modules in $\mathrm{mod}A$. The infinite radical $\mathrm{rad}^\infty(\mathrm{mod}A)$ of $\mathrm{mod}A$ is defined to be the intersection of all powers $\mathrm{rad}^i(\mathrm{mod}A)$, $i \geq 1$, of $\mathrm{rad}(\mathrm{mod}A)$.

The study of $\mathrm{rad}^\infty(\mathrm{mod}A)$, together with the description of the Auslander-Reiten quiver associated to $\mathrm{mod}A$, gives important informations on the complexity of this module category. In particular, it is important to know when $\mathrm{rad}^\infty(\mathrm{mod}A)$ is nilpotent, that is, when there exists an index m such that $(\mathrm{rad}^\infty(\mathrm{mod}A))^m = 0$. If this is the case, we say that the minimum such an index is the *nilpotency index* of $\mathrm{rad}^\infty(\mathrm{mod}A)$ and we indicate it by η_A. In case $\mathrm{rad}^\infty(\mathrm{mod}A)$ is not nilpotent, we shall write $\eta_A = \infty$.

*The author was partially supported by CNPq and FAPESP.
1991 *Mathematics Subject Classification.* 16G20, 16G60, 16G70
Key words and phrases. tilted algebras, infinite radical, module categories.

It has been shown, for instance, in [5] that an algebra A is representation-finite if and only if $(\mathrm{rad}^\infty(\mathrm{mod}A))^2 = 0$, that is, $\eta_A \leq 2$. Recall that an algebra is *representation-finite* provided there exists only finitely many indecomposable objects in mod$A$, up to isomorphism. Otherwise, the algebra is said to be *representation-infinite*. Also, in [6, 7], we have studied the representation-infinite algebras A such that $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 = 0$. Observe that there exist algebras with abitrary nilpotency index [12]. We refer the reader to [10, 12] for more informations on this question.

Here, we are mainly interested in discussing the question of the nilpotency of $\mathrm{rad}^\infty(\mathrm{mod}A)$ for a class of algebras called *tilted algebras*, introduced by Happel and Ringel in [8] (see also [1]). See below for definitions. Our main result can be stated as follows.

Theorem. *Let A be a representation-infinite tilted algebra such that $rad^\infty(modA)$ is nilpotent of index η_A. Then $3 \leq \eta_A \leq 5$. Moreover, if one of the left or the right types of A is empty, then $\eta_A = 3$.*

The notions of left and right types of tilted algebras were introduced in [2] in order to give a better insight of how these algebras were built up. We shall recall their definitions in Section 2 below. The proof of our main result will be given in Section 3. We finish this paper by exhibiting examples showing that all possible values given by the above result ($\eta_A = 3, 4, 5$ or ∞) can occur for a representation-infinite tilted algebra.

1. Preliminaries

1.1. Let k be an algebraically closed field. By algebra is meant a basic and finite dimensional k-algebra. For a given algebra A, we shall denote by mod$A$ the category of all finitely generated right A-modules, and by ind$A$ the full subcategory of mod$A$ with one representative of each isomorphism class of indecomposable A-modules. We shall also keep the notations established in the introduction.

1.2. We shall denote by $\Gamma(\mathrm{mod}A)$ the Auslander-Reiten quiver of A (AR-quiver, for short), and by τ_A the Auslander-Reiten translation in $\Gamma(\mathrm{mod}A)$. We shall now agree to identify the vertices of $\Gamma(\mathrm{mod}A)$ with the corresponding A-modules in $\mathrm{ind}A$. By a component of $\Gamma(\mathrm{mod}A)$ we mean a connected component of $\Gamma(\mathrm{mod}A)$. Observe that if $f\colon X \longrightarrow Y$ is a nonzero morphism with X and Y lying in different components of $\Gamma(\mathrm{mod}A)$, then $f \in \mathrm{rad}^\infty(\mathrm{mod}A)$. We will use this fact along this paper.

We say that a path

$$(*) \qquad X_0 \xrightarrow{f_1} X_1 \xrightarrow{f_2} \cdots \xrightarrow{f_{t-1}} X_{t-1} \xrightarrow{f_t} X_t$$

lies in $\Gamma(\mathrm{mod}A)$ if X_i is indecomposable for each $i = 0, \cdots, t$ and f_i is an irreducible morphism for each $i = 1, \cdots, t$. Also, we say that such a path $(*)$ is *sectional* provided $\tau_A X_i \neq X_{i-2}$, for each $2 \leq i \leq t$.

Let Γ be a component of $\Gamma(\mathrm{mod}A)$. Then, Γ is said to be *regular* if it contains neither a projective module nor an injective module, and *semi-regular* if it does not contain both a projective and an injective module. Further, Γ is *postprojective* (respectively, *preinjective*) if Γ contains no oriented cycles and each module in Γ belongs to the τ_A-orbit of a projective module (respectively, of an injective module). Moreover, the component Γ is said to be *generalized standard* if $\mathrm{rad}^\infty(X, Y) = 0$ for all modules X and Y in Γ . Observe that a postprojective (respectively, a preinjective) component Γ is generalized standard because $\mathrm{rad}^\infty(-, X) = 0$ (respectively, $\mathrm{rad}^\infty(X, -) = 0$) for all $X \in \Gamma$.

For more details on Auslander-Reiten theory we refer to [4, 11].

2. The types of a tilted algebra

2.1. Let Δ be a finite quiver without oriented cycles and consider the path algebra $H = k\Delta$. An H-module T is called *tilting* provided: (i) $\mathrm{Ext}^1_H(T, T) = 0$; and (ii) there exists a short exact sequence $0 \longrightarrow H \longrightarrow T_1 \longrightarrow T_2 \longrightarrow 0$, where T_1 and T_2 belong to the additive subcategory $\mathrm{add}T$ generated by T. A *tilted algebra* is the endomorphism ring of a tilting module over a path algebra as above. The *type* of such tilted algebra A is defined to be the underlined graph $\overline{\Delta}$ of the quiver Δ. It follows from the description of the AR-quiver of a tilted algebra of type Δ that $\Gamma(\mathrm{mod}A)$ has a component, called *connecting*, which contains a section of type Δ, called *complete slice*. This component can be postprojective, preinjective or a directing generalized standard component. In fact, there are at most two of such components and in case there are two, they are, respectively, a postprojective and a preinjective component. In this case the algebra is also called *concealed*. For more details on tilted algebra we refer the reader to [1].

2.2. In [2], in a joint work with Assem, we have introduced the notions of left and right types of a tilted algebra A in order to have some hints of how A is built up. Specifically, we were interested in the study of the homological properties of the indecomposable A-modules in terms of their position in $\Gamma(\mathrm{mod}A)$. We shall use here these types to study those tilted algebras A with $\mathrm{rad}^\infty(\mathrm{mod}A)$ nilpotent.

We define the *left type* of A as follows. If $\Gamma(\mathrm{mod}A)$ has a complete slice in a postprojective component, the left type of A is defined to be the empty graph. Otherwise, $\Gamma(\mathrm{mod}A)$ has a unique connecting component Γ which is not postprojective. If Γ contains no projective module (so that every module in Γ is left stable under the translation τ_A), we define the left type of A to be the type of the tilted algebra A, as defined above. Suppose Γ contains a projective module. Let Σ be the subsection of Γ consisting of the left stable modules M in Γ such that there exists a path in Γ of length at least one from M to some projective, and any such path is sectional. Observe that Σ is not necessarily connected. Then Σ will be called *the left extremal subsection of* A, and its underlying graph $\overline{\Sigma}$ will be called the *left type of* A.

Dually, we define the *right type* of A. For details, we refer to [2].

2.3. Let A be a tilted algebra of type Δ. Suppose its left type Δ_l is non-empty. Then Δ_l has no connected subgraphs of Dynkin type. Indeed, if Δ_l

contains a subgraph Δ' of Dynkin type, then the connecting component Γ would contain a left stable full subquiver of type $-\mathbf{N}\Delta'$. Observe then that the length function restrict to such a subgraph would be an additive function, which is a contradiction. Similarly, there is no connected subgraph of Dynkin type in the right type Δ_r of A.

Clearly, if Δ_l is the empty graph, then $\Gamma(\mathrm{mod}A)$ has a postprojective component containing all projective modules and then, according to [3], $\mathrm{rad}^\infty(-, A_A) = 0$. Dually, if Δ_r is the empty graph, then $\mathrm{rad}^\infty(\mathrm{D}({}_AA), -) = 0$.

2.4. Observe that the notions of left and right types have a close relation with the left and right end algebras, as defined by Kerner [9]. We recall the following lemma from [2].

Lemma. *Let A be a representation-infinite algebra which is tilted but not concealed.*

(a) Each connected component of the left extremal subsection is a complete slice in the connecting component without projective modules of the Auslander-Reiten quiver of a connected component of the left end algebra ${}_\infty A$. In particular, the left type of A equals the type of ${}_\infty A$ as a tilted algebra.

(b) Each connected component of the right extremal subsection is a complete slice in the connecting component without injective modules of the Auslander-Reiten quiver of a connected component of the right end algebra A_∞. In particular, the right type of A equals the type of A_∞ as a tilted algebra.

It follows from [9] that A is tame if and only if A_∞ and ${}_\infty A$ are both tame. For further details on the above notions, we refer the reader to [2, 3, 9].

2.5. The next result follows from [9, 11] (see also [10]).

Proposition. *A tilted algebra A is tame if and only if $rad^{\infty}(modA)$ is nilpotent.*

2.6. We shall now give a necessary condition on the left and the right types for an algebra A to be tame. Recall that a graph is of *wild type* if it is neither Dynkin nor Euclidean.

Proposition. *Let A be a tilted algebra of type Δ. If A is tame, then neither Δ_r nor Δ_l has connected subgraphs of wild type.*

Proof: Without loss of generality, let us suppose that Δ_l has a connected subgraph of wild type. Therefore, by [9](4.1), the left end algebra ${}_{\infty}A$ has a summand A' which is a tilted algebra of wild type, given by a tilting module without preinjective direct summands. By [13](7.6), A' is then of wild type, a contradiction with the fact that A is tame.

□

The next result is a direct consequence of the above considerations.

Corollary. *Let A be a tilted algebra. Then $rad^{\infty}(modA)$ is nilpotent if and only if neither Δ_l nor Δ_r has connected subgraphs of wild type.*

3. The main result

3.1. It follows from the above considerations that the left and the right types of a representation-infinite tilted algebra A are both empty if and only if A is concealed. We then have the following result.

Proposition. *The following are equivalent for a representation-infinite concealed algebra A:*

(a) A is tame;

(b) $(rad^{\infty}(modA))^3 = 0$;

(c) $rad^{\infty}(modA)$ is nilpotent.

Proof: By (2.5), (a) and (c) are equivalent.
(a) $\Rightarrow$ (b) It follows from the description of tame concealed algebra that $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 = 0$.
(b) $\Rightarrow$ (a) Now, if A is of wild type, then $\Gamma(\mathrm{mod}A)$ has a component of type $\mathbf{ZA}_\infty$ (see [11]). Therefore, by [6](2.1), $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 \neq 0$, which contradicts (b).

□

3.2. We shall now prove our main result.

Theorem. *Let A be a representation-infinite tilted algebra such that $rad^\infty(modA)$ is nilpotent of index η_A. Then $3 \leq \eta_A \leq 5$. Moreover, if one of the left or the right types of A is empty, then $\eta_A = 3$.*

Proof: Since A is representation-infinite, we infer by [5] that $(\mathrm{rad}^\infty(\mathrm{mod}A))^2 \neq 0$. Therefore $\eta_A \geq 3$. If both the left and the right types of A are empty, then A is concealed and, by the above lemma, we have that $\eta_A = 3$.
Suppose now that the left type of A is empty but the right type is not. By the above considerations, the right type should be a disjoint union of Euclidean graphs. In particular, A is not concealed and the unique connecting component of $\Gamma(\mathrm{mod}A)$ is a postprojective component Γ. If Γ has no injective modules then, by definition, the right type of A equals its type. Since A is connected, we infer that the type of A is an Euclidean graph. Therefore, by [7](2.1), $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 = 0$.
Suppose now that Γ do have injective modules and let $\Sigma = \Sigma_1 \cup \cdots \cup \Sigma_t$ be the right extremal subsection of A, where for each $1 \leq i \leq t$, Σ_i is connected. Let also $A_\infty = A_1 \times \cdots \times A_t$ be the right end algebra, ordered in such a way that, for each i, $\overline{\Sigma_i}$ is the type of the connected algebra A_i. By hypothesis, each Σ_i is an Euclidean graph and each A_i is a tilted algebra with complete slice in its postprojective component. Therefore, $(\mathrm{rad}^\infty(\mathrm{mod}A_i))^3 = 0$, for each $i = 1, \cdots, t$ (by [7](2.1)). Observe also that $\mathrm{ind}(A_\infty)$ is cofinite in $\mathrm{ind}A$ and

that all the indecomposable A-modules which are not A_∞-modules belong to the postprojective component Γ.

If now $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 \neq 0$, then there exist X, Y, Z and W in $\mathrm{ind}A$ and morphisms f, g, h in $\mathrm{rad}^\infty(\mathrm{mod}A)$,

$$X \xrightarrow{f} Y \xrightarrow{g} Z \xrightarrow{h} W$$

such that $hgf \neq 0$.

Observe first that $W \notin \Gamma$ because $h \in \mathrm{rad}^\infty(\mathrm{mod}A)$ and Γ is postprojective. Hence, $W \in \mathrm{ind}A_i$, for some i. Since hg is a nonzero morphism in $(\mathrm{rad}^\infty(\mathrm{mod}A))^2$, by the description of the Auslander-Reiten quiver of such tilted algebra (see [9]), we infer that Z is a regular A_i-module and Y is either a postprojective A_i-module or a module in $\mathrm{ind}A \backslash \mathrm{ind}A_\infty$. In both cases, $Y \in \Gamma$ and hence $\mathrm{rad}^\infty(-, Y) = 0$. This, however, contradicts our hypothesis that $0 \neq f \in \mathrm{rad}^\infty(X, Y)$. Therefore, in this case, $\eta_A = 3$. Similarly, if the left type of A is a disjoint union of Euclidean graphs and the right type is empty, then $\eta_A = 3$.

It remains to consider the case when both the left and the right types of A are disjoint unions of Euclidean graphs. Clearly, $\Gamma(\mathrm{mod}A)$ has a (unique) connecting component which is neither postprojective nor preinjective.

Write the left end algebra ${}_\infty A$ as $B_1 \times \cdots \times B_t$, where for each $i = 1, \cdots, t$, B_i is a connected tilted algebra with complete slice in the preinjective component, and the right end algebra A_∞ as $C_1 \times \cdots \times C_s$, where for each $j = 1, \cdots, s$, C_j is a connected tilted algebra with complete slice in the postprojective component. Therefore, by [7](2.1), we have

$$(\mathrm{rad}^\infty(\mathrm{mod}B_i))^3 = 0 = (\mathrm{rad}^\infty(\mathrm{mod}C_j))^3$$

for all $i = 1, \cdots, t$ and $j = 1, \cdots, s$. It follows fom this description that $\mathrm{Hom}_A(X, Y) = 0$, in the following cases:

(i) $X \in \mathrm{ind}B_i \setminus \Gamma$ and $Y \in \mathrm{ind}B_j \setminus \Gamma$, with $i \neq j$.

(i') $X \in \mathrm{ind}C_i \setminus \Gamma$ and $Y \in \mathrm{ind}C_j \setminus \Gamma$, with $i \neq j$.

(ii) $X \in \mathrm{ind}C_i$ and $Y \in \mathrm{ind}B_j \setminus \Gamma$, for all i and j.

(ii') $X \in \mathrm{ind}C_i \setminus \Gamma$ and $Y \in \mathrm{ind}B_j$, for all i and j.

(iii) $X \in \Gamma$ and $Y \in \mathrm{ind}B_j \setminus \Gamma$, for all j.

(iii') $X \in \mathrm{ind}C_i \setminus \Gamma$ and $Y \in \Gamma$, for all i.

We refer the reader to [3, 9] for details.

Suppose now that $(\mathrm{rad}^\infty(\mathrm{mod}A))^5 \neq 0$. Then there exists a sequence of morphisms

$$X_1 \xrightarrow{f_1} X_2 \xrightarrow{f_2} X_3 \xrightarrow{f_3} X_4 \xrightarrow{f_4} X_5 \xrightarrow{f_5} X_6$$

with $X_i \in \mathrm{ind}A$, for each $i = 1, \cdots, 6$, $f_j \in \mathrm{rad}^\infty(\mathrm{mod}A)$, for $j = 1, \cdots, 5$, and $f_5 \cdots f_1 \neq 0$. It is not difficult to see that if $X_j \in \mathrm{ind}B_i \setminus \Gamma$, then $j \leq 2$ and if $X_j \in \mathrm{ind}C_i \setminus \Gamma$, then $j \geq 5$. Therefore, X_3 and X_4 belong to Γ, which is a contradiction, because Γ is generalized standard. Therefore, $\eta_A \leq 5$ and the result is proven.

□

4. Examples

4.1. We shall exhibit examples to show that there exist tilted algebras A with $\mathrm{rad}^\infty(\mathrm{mod}A)$ nilpotent of index 3, 4 and 5. Observe also that the examples below show that each of these indexes can occur for tilted algebras with both left and right types being nonempty. On the other hand, any tilted algebra of wild type gives an example of $\eta_A = \infty$ (by (2.5)).

Examples. (a) Let A be the path algebra given by the quiver Δ:

$$\begin{tikzcd} \underset{1}{\bullet} & \underset{2}{\bullet} \arrow[l, shift left, "\alpha"'] \arrow[l, shift right] & \underset{3}{\bullet} \arrow[l, "\beta"'] \end{tikzcd} \qquad \text{with } \alpha\beta = 0$$

The Auslander-Reiten quiver of A consists of: (i) a postprojective component $\mathcal{P}$ corresponding to the Kronecker algebra given by the full subquiver of Δ containing only the vertices 1 and 2; (ii) a family of pairwise orthogonal generalized standard tubes all but one being homogeneous, the exception being a semi-regular tube containing the projective module associated to the vertex 3; (iii) a preinjective component $\mathcal{I}$ containing all the injective modules and a complete slice of type $\tilde{\mathbf{A}}_3$. Observe that: (a) each of the components of $\Gamma(\mathrm{mod}A)$ is generalized standard; (b) the family of tubes is orthogonal; and (c) $\mathrm{rad}^\infty(-,X) = 0$ for each $X \in \mathcal{P}$ and $\mathrm{rad}^\infty(Y,-) = 0$ for each $Y \in \mathcal{I}$. Therefore, $(\mathrm{rad}^\infty(\mathrm{mod}A))^3 = 0$. Since A is representation-infinite, then $\eta_A = 3$. Observe also that the left type of A is empty and the right type equals the type of A, that is, the graph $\tilde{\mathbf{A}}_3$.

(b) Let A be the path algebra given by the quiver Δ:

$$\begin{tikzcd} \underset{1}{\bullet} & \underset{2}{\bullet} \arrow[l, shift left, "\alpha"'] \arrow[l, shift right] & \underset{3}{\bullet} \arrow[l, "\beta"'] & \underset{4}{\bullet} \arrow[l, "\gamma"'] & \underset{5}{\bullet} \arrow[l, shift left, "\delta"'] \arrow[l, shift right] \end{tikzcd}$$

with $\alpha\beta = \beta\gamma = \gamma\delta = 0$. Consider A_1 (and A_2) the k-algebra given by the full subquiver of Δ containing only the vertices $1, 2, 3$ $(3, 4, 5$, respectively) with $\alpha\beta = 0$ $(\gamma\delta = 0$, respectively). Clearly, A_1 is the algebra of the example (a) above and A_2 its opposite algebra. The categories $\mathrm{ind}A_1$ and $\mathrm{ind}A_2$ are naturally embedded into $\mathrm{ind}A$ and, in fact, $\mathrm{ind}A$ is the union of $\mathrm{ind}A_1$ and $\mathrm{ind}A_2$ (the only module which belongs to both $\mathrm{ind}A_1$ and $\mathrm{ind}A_2$ is the simple module S_3 associated to the vertex 3). The description of the AR-quiver of A_1 is given above and the AR-quiver of A_2 is the opposite of the AR-quiver of A_1. They glue together at the commom vertex S_3 to form the AR-quiver of A. Observe, however, that any nonzero morphism $f\colon X \longrightarrow Y$ with $X \in \mathrm{ind}A \setminus \mathrm{ind}A_2$ and $Y \in \mathrm{ind}A \setminus \mathrm{ind}A_1$ factors through S_3 and, therefore, X cannot belong to the postprojective component of $\Gamma(\mathrm{mod}A_1)$ and Y cannot belong to the preinjective component of $\Gamma(\mathrm{mod}A_2)$. Therefore, $\eta_A = 3$. The left and the right types of A are both equal to $\tilde{\mathbf{A}}_3$.

(c) Let A be the path algebra given by the quiver Δ:

$$\begin{tikzcd} \underset{1}{\bullet} & \underset{2}{\bullet} \arrow[l, shift left, "\alpha"'] \arrow[l, shift right] & \underset{3}{\bullet} \arrow[l, "\beta"'] & \underset{4}{\bullet} \arrow[l, shift left, "\gamma"'] \arrow[l, shift right, "\delta"] \end{tikzcd}$$

with $\alpha\beta = \beta\gamma = \beta\delta = 0$. Consider A_1 the k-algebra given by the full subquiver of Δ containing only the vertices $1, 2, 3$ with $\alpha\beta = 0$, and A_2 the Kronecker algebra given by the full subquiver of Δ containing only the vertices 3 and 4. The AR-quiver of A is the glueing of the AR-quivers of A_1 and A_2 by the simple module S_3 at the vertex 3. Using the same sort of arguments as used in (b) above, it is not difficult to see that $\eta_A = 4$.

(d) Let A be the path algebra given by the quiver

$$\underset{1}{\bullet} \overset{\alpha_1}{\underset{\alpha_2}{\leftleftarrows}} \underset{2}{\bullet} \overset{\beta_1}{\underset{\beta_2}{\leftleftarrows}} \underset{3}{\bullet} \qquad \text{with } \alpha_i\beta_j = 0, \quad \forall \ i, j = 1, 2.$$

The AR-quiver of A is the glueing of the two copies of the AR-quivers of Kronecker algebras at the simple module associated to the vertex 2, which belongs to the postprojective component of the Kronecker algebra given by vertices 2 and 3 and the preinjective component of the Kronecker algebra given by vertices 1 and 2. It is not difficult to see that, in this case, $\eta_A = 5$.

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