Principal Typings for Explicit Substitution*

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Outline

Principal Typing

Let $\Gamma \vdash b : A$ be a type judgement in some type system S

- $\tau = (\Gamma, A)$ is a typing of b in S ($S \blacktriangleright b : \tau$).

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- PT property allows *compositional* type inference

Principal Typing vs. Principal Type [Jim96]

Given term b and context Γ, A is a **principal type** of b if it represents any other possible type of b in Γ.

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Question: $y{:}A{\rightarrow}A \vdash \lambda_x.(y\ x) :?$

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Principal Typing vs. Principal Type

	Principal Type	Principal Typing
STLC	✓ [Hi97]	✓ [Wells02]
Hindley/Milner	✓ [DM82]	X [Wells02]
System F	?	X [Wells02]
System $\mathbb{I}$	✓ [KW04]	✓ [KW04]

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Definition (Typing's Partial Order)

Let $\tau \leq_S \tau'$ iff $Terms_S(\tau) \subseteq Terms_S(\tau')$

Principal Typing

Definition (Wells' PT[Wells02])

A typing τ in system S is principal for some term a iff $S \blacktriangleright a{:}\tau$ and $S \blacktriangleright a{:}\tau'$ implies $\tau \leq_S \tau'$.

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Example

For $\tau_1 = (y{:}A{\rightarrow}B, A{\rightarrow}B)$ and $\tau_2 = (y{:}A{\rightarrow}A, A{\rightarrow}A)$ we have $\mathit{Terms}(\tau_1) \subset \mathit{Terms}(\tau_2)$

Principal Typing for STLC

Definition (Hindley's PT [Wells02])

Let $\tau = (\Gamma, B)$, where $\mathrm{TA}_\lambda \blacktriangleright a : \tau$. τ is principal typing of a iff for any typing $\tau' = (\Gamma', B')$ where $\mathrm{TA}_\lambda \blacktriangleright a : \tau'$, then exists some type substitution s such that $s(\Gamma) \subseteq \Gamma'$ and $s(B) = B'$.

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Theorem ([Wells02])

A typing τ is principal according to Wells' PT iff τ is principal according to Hindley's PT.

Simple Type System

Definition (Simple Types and Contexts)

Types $A ::= K \mid A \to A$ **Contexts** $\Gamma ::= nil \mid A.\Gamma$

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- For $\Gamma = A_1.A_2.\cdots.A_m$:
  $\Gamma_{\leq n} = A_1.\cdots.A_n$
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  $\Gamma_{\leq n} = A_1.\cdots.A_n$
  $\Gamma_{\geq n} = A_n.\cdots.A_m$
- $\Gamma_{<n}$ and $\Gamma_{>n}$ defined similar

The System $TA_{\lambda dB}$

Syntax

Terms $\quad a ::= \underline{n} \mid (a\ a) \mid \lambda.a$

Typing Rules

$$A.\Gamma \vdash \underline{1}\ : A\ (Var) \qquad \frac{\Gamma \vdash \underline{n}\ : B}{A.\Gamma \vdash \underline{n+1}\ : B}\ (Varn)$$

$$\frac{A.\Gamma \vdash b : B}{\Gamma \vdash \lambda.b : A \to B}\ (Lambda) \qquad \frac{\Gamma \vdash a : A \to B \quad \Gamma \vdash b : A}{\Gamma \vdash (a\ b) : B}\ (App)$$

PT in $TA_{\lambda dB}$

Lemma (Weakening)

If $\Gamma \vdash a : B$, then $\Gamma.A \vdash a : B$ for any type A.

Definition (PT in $TA_{\lambda dB}$)

Let $\tau = (\Gamma, B)$, where $\mathrm{TA}_{\lambda dB} \blacktriangleright a : \tau$. τ is principal typing of a iff for any typing $\tau' = (\Gamma', B')$ where $\mathrm{TA}_{\lambda dB} \blacktriangleright a : \tau'$, then exists some type substitution s such that $s(\Gamma) = \Gamma'_{\leq|\Gamma|}$ and $s(B) = B'$.

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$TA_{\lambda dB}$ satisfies PT property

Type Inference for $TA_{\lambda dB}$ [AyMu2000]

1st Given term a, let a' be its annoted version.
Ex: for $a = \lambda.(\underline{2}\ \ \underline{1})$, $a' = (\lambda.(\underline{2}\,^{\Gamma_1}_{A_1}\ \ \underline{1}\,^{\Gamma_2}_{A_2})^{\Gamma_3}_{A_3})^{\Gamma_4}_{A_4}$

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2nd Let R_0 be the set of subterms of a'. Start using the type inference algorithm on $\langle R_0, \varnothing \rangle$

3rd When $\langle \varnothing, E \rangle$ is reached, E is the set of equations on type and context variables

4th Use a first order unification algorithm to give you the m.g.u.

Type Inference for $TA_{\lambda dB}$

$$
\begin{array}{ll}
\text{(Var)} & \langle R \cup \{\underline{1}^{\Gamma}_{A}\}, E\rangle \rightarrow \\
 & \qquad \langle R, E \cup \{\Gamma = A.\Gamma'\}\rangle \\
\text{(Varn)} & \langle R \cup \{\underline{n}^{\Gamma}_{A}\}, E\rangle \rightarrow \\
 & \qquad \langle R, E \cup \{\Gamma = A'_1. \cdots .A'_{n-1}.A.\Gamma'\}\rangle \\
\text{(Lambda)} & \langle R \cup \{(\lambda.a^{\Gamma_1}_{A_1})^{\Gamma_2}_{A_2}\}, E\rangle \rightarrow \\
 & \qquad \langle R, E \cup \{A_2 = A^* \rightarrow A_1, \Gamma_1 = A^*.\Gamma_2\}\rangle \\
\text{(App)} & \langle R \cup \{(a^{\Gamma_1}_{A_1}\ b^{\Gamma_2}_{A_2})^{\Gamma_3}_{A_3}\}, E\rangle \rightarrow \\
 & \qquad \langle R, E \cup \{\Gamma_1 = \Gamma_2, \Gamma_2 = \Gamma_3, A_1 = A_2 \rightarrow A_3\}\rangle
\end{array}
$$

Obs: A', A^* and Γ' are fresh variables

The System $TA_{\lambda s_e}$[KR97]

Syntax

Terms $a ::= \underline{n} \,|\, (a\ a) \,|\, \lambda.a \,|\, a\,\sigma^i a \,|\, \varphi^j_k\, a$

Typing Rules

$$\frac{\Gamma_{\leq k}.\Gamma_{\geq k+i} \vdash a : A}{\Gamma \vdash \varphi^i_k\, a : A}\ (Phi) \qquad \frac{\Gamma_{\geq i} \vdash b : B \quad \Gamma_{<i}.B.\Gamma_{\geq i} \vdash a : A}{\Gamma \vdash a\,\sigma^i b : A}\ (Sigma)$$

PT in $TA_{\lambda s_e}$

Definition (PT in $TA_{\lambda s_e}$)

Let $\tau = (\Gamma, B)$, where $\mathrm{TA}_{\lambda s_e} \blacktriangleright a : \tau$. τ is principal typing of a iff for any typing $\tau' = (\Gamma', B')$ where $\mathrm{TA}_{\lambda s_e} \blacktriangleright a : \tau'$, then exists some type substitution s such that $s(\Gamma) = \Gamma'_{\leq |\Gamma|}$ and $s(B) = B'$.

Theorem (Correspondence for $TA_{\lambda s_e}$)

A typing τ is PT of b in $TA_{\lambda s_e}$ iff τ is principal of b according to Wells' PT definition

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\begin{array}{ll}
\text{(Sigma)} & \langle R \cup \{(a^{\Gamma_1}_{A_1}\, \sigma^i\, b^{\Gamma_2}_{A_2})^{\Gamma_3}_{A_3}\}, E\rangle \rightarrow \\
& \qquad \langle R, E \cup \{A_1 {=} A_3, \Gamma_1 {=} A'_1.\cdots.A'_{i-1}.A_2.\Gamma_2, \Gamma_3 {=} A'_1.\cdots.A'_{i-1}.\Gamma_2\}\rangle, \\
\text{(Phi)} & \langle R \cup \{(\varphi^i_k\, a^{\Gamma_1}_{A_1})^{\Gamma_2}_{A_2}\}, E\rangle \rightarrow \\
& \qquad \langle R, E \cup \{A_1 = A_2, \Gamma_2 = A'_1.\cdots.A'_{k+i-1}.\Gamma', \Gamma_1 = A'_1.\cdots.A'_k.\Gamma'\}\rangle,
\end{array}
$$

The System $TA_{\lambda\sigma}$[ACCL91]

Syntax

Terms $a ::= \underline{1} \mid (a\ a) \mid \lambda.a \mid a[s]$ Substitution $s ::= id \mid \uparrow \mid a.s \mid s \circ s$

Typing rules

Terms

$$A.\Gamma \vdash \underline{1} : A \ (var) \qquad \frac{A.\Gamma \vdash b : B}{\Gamma \vdash \lambda.b : A \to B} \ (lambda)$$

$$\frac{\Gamma \vdash a : A \to B \quad \Gamma \vdash b : A}{\Gamma \vdash (a\ b) : B} \ (app) \qquad \frac{\Gamma \vdash s \rhd \Gamma' \quad \Gamma' \vdash a : A}{\Gamma \vdash a[s] : A} \ (clos)$$

Substitutions

$$\Gamma \vdash id \rhd \Gamma \ (id) \qquad A.\Gamma \vdash \uparrow \rhd \Gamma \ (shift)$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash s \rhd \Gamma'}{\Gamma \vdash a.s \rhd A.\Gamma'} \ (cons) \qquad \frac{\Gamma \vdash s'' \rhd \Gamma'' \quad \Gamma'' \vdash s' \rhd \Gamma'}{\Gamma \vdash s' \circ s'' \rhd \Gamma'} \ (comp)$$

PT in $TA_{\lambda\sigma}$

Lemma (Weakening)

If $\Gamma \vdash a : B$, then $\Gamma.A \vdash a : B$ for any type A. Particularly, if $\Gamma \vdash s \triangleright \Gamma'$ then $\Gamma.A \vdash s \triangleright \Gamma'.A$.

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Definition (PT in $TA_{\lambda\sigma}$)

Let $\tau = (\Gamma, \mathbb{T})$, where $\mathrm{TA}_{\lambda\sigma} \blacktriangleright a : \tau$.

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Definition (PT in $TA_{\lambda\sigma}$)

Let $\tau = (\Gamma, \mathbb{T})$, where $\mathrm{TA}_{\lambda\sigma} \blacktriangleright a : \tau.\tau$ is principal typing of a iff for any typing $\tau' = (\Gamma', \mathbb{T}')$ where $\mathrm{TA}_{\lambda\sigma} \blacktriangleright a : \tau'$, then exists some type substitution s such that $s(\Gamma) = \Gamma'_{\leq|\Gamma|}$ and: if $\mathbb{T}$ is a type then $s(\mathbb{T}) = \mathbb{T}'$ otherwise $s(\mathbb{T}) = \mathbb{T}'_{\leq|\mathbb{T}|}$.

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Theorem (Correspondence for $TA_{\lambda\sigma}$)

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Theorem (PT for $TA_{\lambda\sigma}$)

$TA_{\lambda\sigma}$ satisfies PT property

Type Inference for $TA_{\lambda\sigma}$ [Bo95]

(Var)	$\langle R \cup \{\underline{1}^{\Gamma}_{A}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma = A.\Gamma'\}\rangle$
(Lambda)	$\langle R \cup \{(\lambda.a^{\Gamma_1}_{A_1})^{\Gamma_2}_{A_2}\}, E\rangle$	$\rightarrow \langle R, E \cup \{A_2 = A^* \rightarrow A_1, \Gamma_1 = A^*.\Gamma_2\}\rangle$
(App)	$\langle R \cup \{(a^{\Gamma_1}_{A_1}\ b^{\Gamma_2}_{A_2})^{\Gamma_3}_{A_3}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = \Gamma_2, \Gamma_2 = \Gamma_3, A_1 = A_2 \rightarrow A_3\}\rangle$
(Clos)	$\langle R \cup \{(a^{\Gamma_1}_{A_1}[s^{\Gamma_2}_{\Gamma_3}])^{\Gamma_4}_{A_2}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = \Gamma_3, \Gamma_2 = \Gamma_4, A_1 = A_2\}\rangle$
(Id)	$\langle R \cup \{id^{\Gamma_1}_{\Gamma_2}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = \Gamma_2\}\rangle$
(Shift)	$\langle R \cup \{\uparrow^{\Gamma_1}_{\Gamma_2}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = A'.\Gamma_2\}\rangle$
(Cons)	$\langle R \cup \{(a^{\Gamma_1}_{A_1}.s^{\Gamma_2}_{\Gamma_3})^{\Gamma_4}_{\Gamma_5}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = \Gamma_2, \Gamma_2 = \Gamma_4, \Gamma_5 = A_1.\Gamma_3\}\rangle$
(Comp)	$\langle R \cup \{(s^{\Gamma_1}_{\Gamma_2} \circ t^{\Gamma_3}_{\Gamma_4})^{\Gamma_5}_{\Gamma_6}\}, E\rangle$	$\rightarrow \langle R, E \cup \{\Gamma_1 = \Gamma_4, \Gamma_2 = \Gamma_6, \Gamma_3 = \Gamma_5\}\rangle$

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- PT is not a trivial property
- A definition of PT for λ-calculus in de Bruijn notation was proposed and proved correct
- The PT property for $TA_{\lambda dB}$ was proved
- A definition of PT for λs_e and $\lambda\sigma$ were proposed and proved correct
- The PT property were proved using a type inference algorithm for λs_e [AyMu2000] and $\lambda\sigma$ [Bo95].

Further Work

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- Type unification + substitution calls for *expansion variables*

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