On Artin's conjecture, I: Systems of diagonal forms

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1. Introduction. As a special case of a well-known conjecture of Artin, it is expected that a system of R additive forms of degree k, say

$$\sum_{i=1}^{N} a_{ij}x_i^k = 0 \quad (1 \le j \le R) \tag{1}$$

with integer coefficients a_{ij}, has a *non-trivial* solution in $\mathbb{Q}_p$ for all primes p whenever

$$N > Rk^2. \tag{2}$$

Here we adopt the convention that a solution of (1) is non-trivial if not all the x_i are 0. To date, this has been verified only when $R = 1$ by Davenport and Lewis [4], and for odd k when $R = 2$ by Davenport and Lewis [5]. For larger values of R, and in particular when k is even, more severe conditions on N are required to assure the existence of p-adic solutions of (1) for all primes p. In another important contribution, Davenport and Lewis [6] showed that the conditions

$$N \ge 9R^2k\log(3Rk) \;\; (k \text{ odd}), \qquad N \ge 48R^2k^3\log(3Rk^2) \;\; (k > 2 \text{ even})$$

are sufficient. There have been a number of refinements of these results. Schmidt [13] obtained $N \gg R^2k^3\log k$, and Low, Pitman and Wolff [10] improved the work of Davenport and Lewis by showing the weaker constraints

$$N \ge 2R^2k\log k \;\; (k \gg 1 \text{ odd}), \qquad N \ge 48Rk^3\log(3Rk^2) \;\; (k > 2)$$

to be sufficient for p-adic solubility of (1).

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A noticeable feature of these results is that for even k one always encounters a factor $k^3 \log k$, in spite of the expected k^2 in (2). In this paper we show that one can reach the expected order of magnitude k^2.

THEOREM 1. *Let $k \geq 3$ and $R \geq 3$. Then the system of equations* (1) *has a non-trivial solution in $\mathbb{Q}_p$ for all primes p provided that*

$$N \geq R^3 k^2$$

unless $R = 3$ and k is a power of 2 in which case the condition on N has to be replaced by $N \geq 36k^2$.

For small values of R or k our analysis can be considerably refined. We shall discuss in greater detail the case of pairs of equations $R = 2$. In the light of the aforementioned result of Davenport and Lewis, only even k deserve attention. Davenport and Lewis [5] showed that for even k, the pair of equations

$$\sum_{i=1}^{N} a_i x_i^k = \sum_{i=1}^{N} b_i x_i^k = 0 \tag{3}$$

with $a_i, b_i \in \mathbb{Z}$ has a non-trivial p-adic solution for all primes p when $N \geq 7k^3$, and this remained unimproved until very recently when Godinho [9] obtained bounds on N which are dependent on the prime factorisation of the degree. However, it does not follow from the work of [9] that a condition like $N \geq Ck^2$ with some constant C suffices to guarantee solubility of (3) in all $\mathbb{Q}_p$. Our second theorem provides such a bound with $C = 16$.

THEOREM 2. *If k is of the form*

$$k = 2 \cdot 5^\tau \quad \text{or} \quad k = (p-1)p^\tau \text{for some prime } p > 2 \tag{4}$$

then the pair (3) *has a non-trivial solution in all p-adic fields whenever*

$$N \geq 6k(k-1).$$

If k is not of the form (4) *but*

$$k = k_0 2^\tau \quad \text{with } k_0 = 1 \text{ or } 3 \text{ or } 5 \text{ or } 7 \tag{5}$$

then the same conclusion holds if $N \geq 16k^2 k_0^{-1} - 4k$.
If k is neither of the form (4) *or* (5) *but takes the shape*

$$k = 2p^\tau(p-1), \tag{6}$$

then for $N \geq 3k(k-2)$ *the equations* (3) *have a non-trivial solution in all* p*-adic fields. When* k *is neither of the forms* (4), (5) *or* (6), *then the condition* $N \geq 2k^2 + 1$ *suffices.*

Godinho [8] considered pairs of degree $k = 2^\tau$, and obtained the slightly superior sufficient condition

$$N \geq 16k^2 - 26k + 1.$$

Our approach follows earlier work in all preparatory steps. We shall begin with the p-normalisation process. This amounts to finding a system of equations (1) which is equivalent to the given one but has additional properties to faciliate the later analysis. Then we reduce the problem to finding a non-singular solution to an auxiliary congruence. This part is standard and will be quoted from the literature in §2. We then dismiss primes not dividing the degree in §3 by a simple application of Chevalley's theorem. For primes dividing the degree, congruences to prime power modulus have to be considered, and in §4 we apply a result of Olson [12] in combinatorial group theory to solve them. Theorem 1 will then be immediate, and in the last section Theorem 2 will be deduced by a finer analysis, but based on the same ideas.

Olson's powerful theorem provides, in a certain sense, a suitable substitute for Chevalley's theorem when prime power moduli occur. This is our main source for improvement. Baker and Schmidt [2] have also used Olson's theorem in related problems, but its use for the present problem appears to be new.

We mention in passing that for very large primes p the number of variables required for the existence of p-adic solutions reduces to $N > 2Rk$. See Atkinson, Brüdern and Cook [1] and Meir [11] for work in this direction.

2. Normalisation. In this section we briefly recall the concept of p-normalisation introduced by Davenport and Lewis [6]. Let $A = (a_{ij})$ be the matrix of coefficients of (1), and write $\mathbf{a}_j = (a_{ij})_{1 \leq i \leq R}$ to denote the j-th column of A. Let

$$\theta(A) = \prod_{1 \leq i_1 < i_2 < \ldots < i_R \leq N} \det(\mathbf{a}_{i_1} \mathbf{a}_{i_2} \ldots \mathbf{a}_{i_R}).$$

For a fixed prime p, suppose we wish to investigate whether or not the system (1) admits a non-trivial p-adic solution. Then, in (1) we may replace the original equations by any R independent linear combinations thereof (this corresponds to row operations applied to A). Moreover, since $\mathbb{Q}_p$ is a field of characteristic 0, we may replace a variable x_i with $p^\nu x_i$, for any $\nu \in \mathbb{N}$, and then divide the resulting equations by any power of p which divides all coefficients. Two systems of equations (1) are called p-equivalent if one can be obtained from the other by a finite succession of these processes. A system (1) is called p-normalised if $\theta(A) \neq 0$ and the power of p dividing $\theta(A)$ is minimal among all systems which are p-equivalent to the given one.

LEMMA 1. *Let $k \geq 2$, $N > R$ and suppose that* (1) *admits non-trivial p-adic solutions for all p-normalised systems. Then,* (1) *has non-trivial p-adic solutions for any choice of integer coefficients.*

Proof. See Davenport and Lewis [6], §4.

Following Davenport and Lewis [5] in spirit, we say that the variable x_i is at level l if $p^l | \mathbf{a}_i$ but $p^{l+1} \not| \mathbf{a}_i$. If a system is p-normalised, all variables are at a level less than k. To see this suppose that x_i is at level $l \geq k$. Then $p^{-k}\mathbf{a}_i$ has integral components, and therefore the substition $x_i' = px_i$ changes the θ-value of A by a factor p^{-Mk} for some $M > 0$.

Suppose that (1) is p-normalised, and let n denote the number of variables at level 0. By Lemma 11 of Davenport and Lewis [6], one has

$$n \geq N/k. \tag{7}$$

We may renumber the variables of (1) to arrange that $x_1, x_2, \ldots, x_n$ are the variables at level 0, and we denote the submatrix of A, consisting of the first n columns, by A_0. We consider A_0 as a matrix with coefficients in the finite field $\mathbb{F}_p$ of p elements. For $1 \leq \nu \leq R$ the invariant q_ν is defined as the minimum number of non-zero columns in any ν linear combinations of the rows of A_0 which are independent over $\mathbb{F}_p$. Again by Lemma 11 of Davenport and Lewis [6],

$$q_\nu \geq \nu N/(Rk) \quad (1 \leq \nu \leq R). \tag{8}$$

Now let $\mu(d)$ be the maximal number of columns of A_0 which lie in a d-dimensional linear subspace of $\mathbb{F}_p^R$. Then

$$q_\nu + \mu(R - \nu) = n \tag{9}$$

for $1 \leq \nu \leq R$. Low, Pitman and Wolff [10] observed that the invariants $\mu(d)$ control non-singular $R \times R$ submatrices of A_0. From a combinatorial result on matroids they deduced that for any $t \in \mathbb{N}$, the matrix A_0 will contain t disjoint $R \times R$ submatrices with determinant not divisible by p, if and only if,

$$n - \mu(d) \geq t(R - d) \quad \text{for all } 0 \leq d \leq R$$

(this is Low, Pitman and Wolff [10], Lemma 1). By (9), this is equivalent with $q_\nu \geq t\nu$ for $1 \leq \nu \leq R$, and by (8), we may conclude as follows.

LEMMA 2. *Suppose that* (1) *is p-normalised and has n variables at level* 0. *Then the* $n \times R$ *matrix* A_0 *contains at least* $[N/(Rk)]$ *disjoint* $R \times R$ *submatrices with determinant not divisible by p.*

As a final preparation for our approach to the theorems, we reduce the question of p-adic solubility to congruences. Let $\gamma \geq 1$. A solution of the system of congruences

$$\sum_{i=1}^{N} a_{ij} x_i^k \equiv 0 \bmod p^\gamma \quad (1 \leq j \leq R) \tag{10}$$

is called non-singular if there are $i_1, \ldots, i_R$ with

$$x_{i_1} x_{i_2} \ldots x_{i_R} \det(\mathbf{a}_{i_1} \ldots \mathbf{a}_{i_R}) \not\equiv 0 \bmod p.$$

For a given k we define τ via $p^\tau \parallel k$ and write

$$\gamma = \gamma(k;p) = \begin{cases} \tau + 2 & \text{if } p = 2, \tau > 0, \\ \tau + 1 & \text{otherwise.} \end{cases} \tag{11}$$

LEMMA 3. *Suppose that the congruences* (10) *have a non-singular solution when* γ *is given by* (11). *Then the equations* (1) *have a non-trivial solution in* $\mathbb{Q}_p$.

This is a version of Hensel's Lemma, see Davenport and Lewis [6], Lemma 9.

3. Congruences modulo primes. In this section we provide non-singular solutions to the system of congruences when $\gamma = 1$, and as a corollary obtain a version of Theorem 1 for all primes $p \nmid k$. We begin by recalling a classic result of Chevalley [3].

LEMMA 4. *Let $k \geq 1$ and p be a prime. Let $\delta = (k, p-1)$. Let c_{ij} be any integers and $m > R\delta$. Then the system of congruences*

$$\sum_{i=1}^{m} c_{ij} x_i^k \equiv 0 \bmod p \quad (1 \leq j \leq R) \tag{12}$$

admits a primitive solution.

Recall that a solution of congruences is called primitive if not all its coordinates are divisible by p.

Simple examples show that all solutions of (12) may be singular. However, the following corollary yields non-singular solutions.

LEMMA 5. *Let k, p, δ, c_{ij} be as in the previous Lemma. Suppose that*

$$m \geq R^2(\delta - 1) + 2R, \tag{13}$$

and that the $m \times R$ matrix (c_{ij}) contains $R(\delta-1)+2$ disjoint $R \times R$ matrices with determinant not divisible by p. Then the system of congruences (12) *admits a non-singular solution.*

Proof. By renumbering the columns $\mathbf{c}_i$ of (c_{ij}) we may suppose that the $R \times R$ matrices $(\mathbf{c}_{lR+1} \ldots \mathbf{c}_{(l+1)R})$ for $0 \leq l \leq R(\delta - 1) + 1$ are all non-singular. We may now assume that $m = R^2(\delta - 1) + 2R$ (take $x_i = 0$ for $i > R^2(\delta - 1) + 2R$ otherwise). Put

$$b_{lj} = \sum_{i=lR+1}^{(l+1)R} c_{ij} \quad (0 \leq l \leq R(\delta - 1)) \tag{14}$$

and consider the system of congruences

$$\sum_{l=0}^{R(\delta-1)} b_{lj} y_l^k + \sum_{i=m-R+1}^{m} c_{ij} x_i^k \equiv 0 \bmod p \quad (1 \leq j \leq R) \tag{15}$$

This involves $R\delta + 1$ variables and therefore has a primitive solution by Lemma 4. Since the columns $\mathbf{c}_{m-R+1}, \ldots, \mathbf{c}_m$ are linearly independent (mod p), any primitive solution of (15) must have at least one of the y_l not divisible by p. By taking

$$x_i = y_l \quad \text{for} \quad lR < i \leq (l+1)R,\ 0 \leq l \leq R(\delta - 1)$$

we obtain a solution of (12) which is non-singular.

It is now easy to deduce a result on p-adic solubility when $p \not| k$. In this case Lemma 3 is applicable with $\gamma = 1$. By Lemmas 2 and 5 we may conclude as follows.

THEOREM 3. *Let p be a prime, $p \not| k$ and $N \geq Rk(R(k, p-1) - R + 2)$. Then the system of equations* (1) *admits a non-trivial solution in* $\mathbb{Q}_p$.

A result very similar to this occurs *inter alia* in Davenport and Lewis [6]. We have preferred to present the above approach which is somewhat different from previous techniques, and can serve as a model for the more original arguments in the next section. It would be very interesting to weaken the condition (13). When $R = 2$, Davenport and Lewis [5] established the following result.

LEMMA 6. *Let $k \geq 2$, p be prime and $\delta = (k, p-1)$. Let $a_i, b_i \in \mathbb{Z}$ $(1 \leq i \leq m)$ and suppose that $m \geq 2\delta + 1$. Further suppose that any linear combination of the rows $(a_i), (b_i)$ with coefficients not both divisible by p contains at least $\delta + 1$ entries not divisible by p. Then the pair of congruences*

$$\sum_{i=1}^{m} a_i x_i^k \equiv \sum_{i=1}^{m} b_i x_i^k \equiv 0 \bmod p$$

has a non-singular solution.

One may easily deduce that for $R = 2$, the equations (1) have a non-trivial solution in $\mathbb{Q}_p$ when $N \geq 2k^2+1$ and $p \not| k$. However, when $R = 3$, the natural generalisation of Lemma 6, with $m \geq 3\delta + 1$, $q_\nu \geq \nu\delta + 1$ $(\nu = 1, 2)$, is false. See Davenport and Lewis [7], p. 344, for details and further comments.

4. Primes dividing the degree. We complete the proof of Theorem 1 in this section by considering primes $p|k$. In this case we shall solve the congruences (10) with the aid of combinatorial group theory. We begin with recalling a result of Olson [12]. Let G be a an (additive) finite abelian p-group. Then G is isomorphic to

$$\mathbb{Z}/p^{e_1}\mathbb{Z} \times \ldots \times \mathbb{Z}/p^{e_r}\mathbb{Z}$$

for suitable $e_j \in \mathbb{N}$. If $g_1, \ldots, g_s \in G$ and

$$s > \sum_{j=1}^{r} (p^{e_j} - 1)$$

then, by Olson's theorem, there are $\epsilon_i \in \{0, 1\}$, not all 0, with $\epsilon_1 g_1 + \ldots + \epsilon_s g_s = 0$. We need this result only when all e_j are equal, and reformulate it in the language of congruences when $G = (\mathbb{Z}/p^t\mathbb{Z})^R$.

LEMMA 7. *Let $b_{ij} \in \mathbb{Z}$ $(1 \leq i \leq s, 1 \leq j \leq R)$. Let p be a prime and $t \geq 1$. Then, provided that $s > R(p^t - 1)$, there are $\epsilon_i \in \{0, 1\}$, not all 0, such that*

$$\sum_{i=1}^{s} \epsilon_i b_{ij} \equiv 0 \bmod p^t \quad (1 \leq j \leq R).$$

It is now easy to modify the arguments of the previous section to establish the following result.

THEOREM 4. *Let p be a prime with $p|k$ and define γ by (11). Then, provided that*

$$N \geq Rk(R(p^\gamma - 2) + 2) \tag{16}$$

the equations (1) have a non-trivial solution in $\mathbb{Q}_p$.

Proof. By Lemma 2, we may suppose that the variables x_i with $1 \leq i \leq n$ are at level 0 where $n \geq R^2(p^\gamma - 2) + 2R$, and that the matrices

$$(\mathbf{a}_{lR+1} \ldots \mathbf{a}_{(l+1)R}) \tag{17}$$

with $0 \leq l \leq R(p^\gamma - 2) + 1$ are all non-singular (mod p). We define b_{lj} by (14). The system of congruences

$$\sum_{l=0}^{R(p^\gamma-2)} b_{lj} y_l^k + \sum_{i=R^2(p^\gamma-2)+R+1}^{R^2(p^\gamma-2)+2R} a_{ij} x_i^k \equiv 0 \bmod p^\gamma \quad (1 \leq j \leq R)$$

involves $R(p^\gamma - 1) + 1$ variables and therefore has, by Lemma 7, a solution with $y_l \in \{0, 1\}$, $x_i \in \{0, 1\}$, not all 0. As in the previous section we see that at least one y_l is non-zero, and this yields a non-singular solution of the system

$$\sum_{i=1}^{R^2(p^\gamma-2)+2R} a_{ij}x_i^k \equiv 0 \bmod p^\gamma \quad (1 \le j \le R),$$

as required in Lemma 3 to complete the proof of the theorem.

We have included Theorem 4 mainly for use with very small primes where it proves to be highly effective. It also has a certain interest on its own right. If the prime factorisation of k is "neat enough", then one may deduce from (16) much better bounds then available from Theorem 1. For example, if $p^\gamma \le k$ holds for all $p|k$ then (1) is soluble in all $\mathbb{Q}_p$ whenever $N \ge R^2k^2$. However, one cannot expect to deduce Theorem 1 from Theorem 4. If $k = 2p$ for some odd prime p, then $\gamma = 2$ and in (16) about $\frac{1}{2}R^2k^3$ variables are required. Fortunately there is an alternative approach through *contractions*, a term coined by Davenport and Lewis [4]. This will keep the bounds quadratic in k, but at the price of an extra factor R.

THEOREM 5. *Let $p \ne 2$ and suppose that $p^\tau \parallel k$, $\delta = (k, p-1)$. Then, provided that*

$$N \ge Rk(R(\delta-1)+2)(R(p^\tau-1)+1),$$

the equations (1) *have a non-trivial p-adic solution in $\mathbb{Q}_p$.*

We begin by describing the contraction argument. Suppose that the system (1) is p-normalised and that the matrix A_0 of the columns at level 0 contains T disjoint blocks of $R \times R$ submatrices which are non-singular (mod p). Put $H = R(\delta-1)+2$ and suppose that $T = Ht$ with some $t \in \mathbb{N}$. We may then assume that the matrices (17) with $0 \le l \le T-1$ are all non-singular (mod p). By Lemma 5, the congruences

$$\sum_{hHR<i\le(h+1)HR} a_{ij}u_i^k \equiv 0 \bmod p \quad (1 \le j \le R) \tag{18}$$

have a non-singular solution for any choice of $0 \le h \le t-1$. We then write

$$\sum_{hHR<i\le(h+1)HR} a_{ij}u_i^k = pb_{hj} \tag{19}$$

with integers b_{hj}, and consider the congruences

$$\sum_{h=0}^{t-1} b_{hj}\epsilon_h \equiv 0 \bmod p^\tau \quad (1 \le j \le R), \tag{20}$$

to be solved with $\epsilon_h \in \{0,1\}$. If $t \ge R(p^\tau - 1) + 1$ such a solution exists with not all $\epsilon_h = 0$. By suitable renumbering, we may assume that (20) holds with $\epsilon_h = 1$ for $0 \le h \le H_1$ and $\epsilon_h = 0$ for $H_1 < h \le t-1$, with some $H_1 \ge 0$. From (19) we now deduce that

$$\sum_{h=0}^{H_1} \sum_{hHR < i \le (h+1)HR} a_{ij} u_i^k \equiv 0 \bmod p^{\tau+1},$$

and the solution is non-singular by construction. For $p \ne 2$ we have $\gamma = \tau + 1$, and this establishes the non-singular solubility of (10).

Theorem 5 is now available. Take $t = R(p^\tau - 1) + 1$ and $T = Ht$ as above. If $N \ge RkT$, the matrix A_0 will contain the required T disjoint non-singular blocks for any p-normalised system (1). Theorem 5 now follows from Lemmas 1 and 3.

When $R = 2$, the result can be refined by injecting Lemma 6 in place of Lemma 5 in the above argument. If $m = 2\delta + 2$ and the matrix $\binom{a_i}{b_i}_{i \le m}$ splits into $\delta + 1$ disjoint 2×2 matrices which are non-singular (mod p), then for any λ, μ not both divisible by p, the numbers $\lambda a_i + \mu b_i$ will contain at least $\delta + 1$ numbers not divisible by p. Hence, the congruences in Lemma 6 have a non-singular solution. Consequently, in the preceeding argument, we may take $H = \delta + 1$, and then proceed as before to deduce the following.

THEOREM 6. *Let p, k and δ be as in Theorem 5. Then the pair of equations (3) with integer coefficients admits a non-trivial p-adic solution provided that*

$$N \ge 2k(\delta + 1)(2p^\tau - 1).$$

Theorem 1 is now a simple corollary. For all primes $p \nmid k$ the required conclusion is immediate from Theorem 3, and when $p | k$, $p \ne 2$, Theorem 5 yields the required result. When $p = 2$ and $2 | k$, we write $k = 2^\tau k_0$ with odd k_0. By Theorem 4, the equations (1) have a non-trivial 2-adic solution whenever

$$N \ge 2Rk(R(2^{\tau+1} - 1) + 1),$$

which is more than required unless $R = 3$ and $k_0 = 1$ in which case the condition $N \geq 36k^2$ certainly suffices.

5. Pairs of equations. We shall now deduce Theorem 2. We may suppose that k is even since otherwise $N \geq 2k^2 + 1$ suffices by Theorem 1 of Davenport and Lewis [5]. When k is even, Davenport and Lewis [5] have shown the following.

LEMMA 8. *Let k be even and suppose that $p^\tau \parallel k$, $\delta = (k, p-1)$. If $\tau = 0$ and $N \geq 2k^2 + 1$, the pair of equations* (3) *has a non-trivial p-adic solution. If $\delta < \frac{1}{2}(p-1)$, or $\delta = \frac{1}{2}(p-1) \geq 3$, then again the equations* (3) *have a non-trivial p-adic solution whenever $N \geq 2k^2 + 1$.*

Proof. The first statement follows from Lemma 6. For the second statement, see Davenport and Lewis [5], sections 6 and 7.

It now remains to discuss the following cases:

$$p|k, \quad \delta = p - 1 \tag{21}$$

and

$$p|k, \quad \delta = \frac{1}{2}(p-1) < 3. \tag{22}$$

Note that (22) implies that $p < 7$, and since k is even, the cases $p = 2$ and 3 cannot occur. Hence (22) occurs only for $p = 5$, when $5|k$ and $\delta = (4, k) = 2$. This means $2 \parallel k$, and we may write $k = 2 \cdot 5^\tau k_0$ with $(10, k_0) = 1$. In this particular case, Theorem 6 yields 5-adic solubility for

$$N \geq 6k(kk_0^{-1} - 1).$$

If $k_0 = 1$, this is one of the exceptional cases in Theorem 2, and $k_0 > 1$ implies $k_0 \geq 3$. In this last case, $N \geq 2k^2 + 1$ will suffice.

We can now concentrate on the case (21). We can then write $k = p^\tau(p-1)k_0$. First suppose that $p \neq 2$. Then, by Theorem 6, we see that the equations (3) have a p-adic solution whenever

$$N \geq 2kp(2p^\tau - 1) = \frac{4p}{p-1}\frac{k^2}{k_0} - 2kp. \tag{23}$$

Note that $p/(p-1) \leq 3/2$. Hence, if $k_0 \geq 3$, then $N \geq 2k^2 + 1$ will certainly suffices to to guarantee p-adic solubility of (3). When $k_0 = 2$, we use $p \geq 3$

in (23) to see that $N \geq 3k(k-2)$ suffices. Finally, when $k_0 = 1$, the same reasoning shows that $N \geq 6k(k-1)$ is enough to guarantee solubility of (3) in $\mathbb{Q}_p$.

Finally we discuss 2-adic solubility. Here we write $k = 2^\tau k_0$ with odd k_0 and apply Theorem 4 with $R = p = 2$. This shows that (3) admits non-trivial p-adic solutions whenever $N \geq 16k^2 k_0^{-1} - 4k$.

Theorem 2 is now immediate.

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